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NONLINEAR GRAVIDYNAMICS: ENERGY-MOMENTUM TENSOR OF COLLAPSAR FIELD

VLADIMIR V. SOKOLOV

Department of Relativistic Astrophysics of the Special Astrophysical Observatory of the Russian Academy of Science, Stavropol Territory, Russia

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Abstract. Within the scope of a theoretical scheme treating gravitational interaction consistently as a dynamical (gauge) field in flat space-time, an expression was obtained for the density of gravitational field energy-momentum-tensions in vacuum around a collapsed object (collapsar). The case was studied of an interacting static spherically-symmetric field of the collapsar in vacuum with taking into account all possible components (spin states of virtual gravitons) contributions into energy for the symmetric second rank tensor ψ_{ik}. The radius of a sphere filled by matter for the collapsar with mass M can reach values up to GM/c^2.

1. Introduction

This paper continues the paper (Sokolov, 1992) which began the study of physical properties of objects with extremely strong gravitational fields on their surfaces, i.e., on collapsars from the standpoint of the consistent dynamical description of gravitational interaction. The main purpose of the paper is the solution of the problem in energy-momentum-tensions of the collapsar field when this gravitational field is strong.

In what follows, the question is basically on strong fields of objects with masses of the order of several solar masses - i.e., on the collapsars with stellar masses. Macroscopic average densities of such objects are at least nuclear ones and achieve supernuclear density. Correspondingly, gravitational field energy densities on such collapsar surface may become equal or even more than $\rho_{nucl} \cdot c^2$. If we take as an example such object as a neutron star with the mass $M = 1.44\ M_{\odot}$ and with the radius 10 km and evaluate the gravitational field energy density on the neutron star surface by the formula $(\nabla \varphi_N)^2/8\pi G$, then the energy connected with the field alone turns out to be enormous and approximates to the rest energy of the neutron star matter itself.

Can the field energy density greater than or of the order of $\rho_{nucl} \cdot c^2$ be non-localizable? In the context of a purely geometrical interpretation of gravitation field the answer to this question is known for a long time: the field energy is nonlocalizable even in such a case. The consistent dynamical formulation of the gravitational interaction theory proceeds as a matter of fact from the notion that every cubic centimeter of space contains a completely determined quantity of gravitational field energy-momentum-tensions. Of course, the final answer in this debate will be obtained as a result of a space regions observation with the strong field. Just in the context of possible new observational consequences I continue to formulate here the collapsar problem in gravidynamics.

In particular, as was noted in the previous paper (Sokolov, 1992), the collapsars can have surfaces always. But for a rigorous proof of that, one must first of all clear up completely the question on the gravitational field energy-momentum-tensor (EMT).

In connection with the foregoing I shall emphasize through all the paper the characteristic features of formulation of the collapsar field problem in direct connection with the field energy problem. For all this I consistently adhere to the theoretical scheme in which the gravitational interaction, equally with other ones, is considered as a dynamic field plunged into flat space-time. I note here once more that in such a case one may accept at once that energy is localizable, positive and is understood in the same sense as in any other field theory, in particular, in the classical electrodynamics. I am not going to prove here especially the justice of such natural demands (axioms) in the dynamical field theory. It is more interesting to elucidate what observational consequences their fulfilment brings to, if the axioms are really true.

We begin here (in the Introduction) with the most important, principle aspects underlying the approach developed by us (Sokolov, 1992; Sokolov and Baryshev, 1980; Baryshev and Sokolov, 1984) to the description of gravitational interaction. The term 'gravidynamics' (GD) used below (and also frequently used by specialists in gravitation) seems to me the best one reflecting the features of our approach.

As was shown in detail in the previous paper (Sokolov, 1992), the field energy density near a gravitating body with the mass M at the distance r from its centre, can be given as a matter of fact by

$$\theta_{00} = (\nabla\, \varphi_N)^2/8\pi G, \quad \text{where} \quad \varphi_N = -GM/r\,,$$

if at its deduction one take into account the fulfillment of three main conditions for the field EMT:

$$\theta_{00} \geq 0\,, \quad \theta_{ik} = \theta_{ki}\,, \quad \theta^{ik}\eta_{ik} = 0\,, \quad i,k = 0,1,2,3.$$

Where η_{ik} = diag(+1, −1 , −1 , −1) is Minkowsky's metric tensor. I emphasize at once that in GD you *may* use only this always the same constant metric at the *consistent* dynamic description of gravitational interaction you may do that in the case of all other interactions.

To understand correctly following sections one should not forget and consistently adhere to the concept (which became already a common place) that, in the relativistic field theory, one may not ascribe straight away some finite dimension both to test particles and to particles ('matter') - sources of the field. I.e., at the formulation of a gravitational interaction relativistic theory (like for all modern field theories) it is more logical at least at the beginning to proceed from the fact that the right-hand sides of corresponding field equations can contain a point source or a system ($\sum_a m_a$) of point sources: i.e., a gravitating 'body'. In particular, every macroscopic region, which the real gravitating body is formed of, can be presented as a 'point' with the mass m_a. These regions are the 'points' between which mainly only gravitation acts. In GD the fundamental Special Relativity concept about interacting points (usual for local theory) is used as an *initial* concept of 'gravitational charges'. Of course, a question arises on the justice of these idealized notions for the macroscopic theory which the gravitation theory is. As we will see from the following, an exhaustive answer to such a question can be obtained ultimately as a full solution of the problem of physical properties of GD collapsars.

As it was in the paper by Sokolov (1992), one can begin again to investigate first of all what is a unit 'elementary' point source with the mass *M*. In a sense, the collapsar itself is such an 'elementary' object by analogy with the elementary point charge - electron - in electrodynamics (ED). In GD around any spherically-symmetric distributed mass there is also the field with the energy density θ_{00} - 'a coat' of virtual gravitons. At some distance from the centre of the object in 'vacuum' (i.e., out of the sphere filled by matter) θ_{00} can turn out to become of the order of the average rest energy density of the collapsar - i.e., of the system 'matter + field'

$$\theta_{00} = GM^2/8\pi r^4 \simeq Mc^2/r^3 .$$

Thus, in GD which is a macroscopic theory a question arises, which we meet one way or another in the classical ED, quantum ED (QED), quantum chromodynamics (QCD), etc.: where in this case and in what form is the mass of such a 'point' object concentrated? What is the collapsar *rest mass* in general in GD?

Such problems inevitably arise in GD also when the distance from the collapsar centre become of order of the gravitational radius GM/c^2 of this 'point' object. Here it turns out that the quantity GM/c^2 is a direct analogue of the classical electron radius. Just as in the classical ED, one can show that until the point distribution ($\sum_a m_a$) compressed to a dimension when distances between the points become of the order of Gm_a/c^2, we deal with a theory quite analogous to the classical (and linear) ED.

From the foregoing it is clear that in GD the 'point' source with the mass *M* is in fact 'something' having a finite dimension greater-than or ~ GM/c^2. At $r >> GM/c^2$ one can be uninterested at all in the structure of such an 'elementary' point object. But then the mass of the 'coat' of the virtual gravitons must be automatically included in the source mass. Thus, that part of the theory, in which the notion of the point gravitating object with the mass *M* is true, can be described by linear equations.

I.e., in this linear approximation GD when gravitating sources may be quite assumed to be point structureless objects for which the mass conservation is still fulfilled to a high precision as

$$\left(\mu\frac{dx^k}{dt}\right)_{,k}=0\,,\quad \text{where}\quad \mu=\sum_a m_a\delta(\mathbf{r}-\mathbf{r}_a),$$

one can, by using usual rules, put down the Lagrangian of a symmetric tensor field ψ_{ik} interacting with its sources. I emphasize that the consistent dynamic interpretation of the field equations fitting to this Lagrangian rely on fact that potentials of the field ψ_{ik} (just as in ED) should be understood absolutely independently of the chosen metrics η_{ik}. In particular, in GD it is senseless to speak about the condition $\psi_{ik} << \eta_{ik}$. Like the vector 4-potential in ED, ψ_{ik} can be of any value in virtue of indeterminancy of

$$\psi_{ik} \rightarrow \psi_{ik} + A_{i,k} + A_{k,i} + \Lambda_{,ik}$$

This transformation for ψ_{ik} is the gauge one with an arbitrary 4-vector A_i and arbitrary 4-scalar Λ. It can be connected as usual with the masslessness of the tensor field ψ_{ik}. Ultimately it is not the Lorentz invariance demand alone but the demand of the gauge invariance in the linear approximation of GD also which determines both the field Lagrangian and the field equations in a unique fashion.

In the linear approximation the interaction of the field ψ_{ik} with its sources is described by the term $f\psi_{ik}T^{ik}$ of the Lagrangian (f is the coupling constant). If adhering consistently to the structurelessness (or the pointness) of particles with the rest mass $m_a \neq 0$ interacting with gravitation, then to describe the substance which is usually called the 'matter' we must choose as *the point of departure* the tensor (EMT) of the system of structureless point objects-particles:

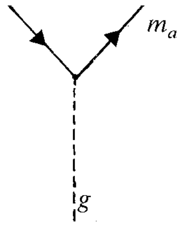
$$T^{ik} = \mu c u^i u^k (\mathrm{d}s/\mathrm{d}t), \quad \text{where}\ \ \mathrm{d}s = c\,\mathrm{d}t\,\sqrt{1-\mathrm{v}^2/c^2}$$

where u^i is the velocity 4-vector and v is the usual velocity of particles. In the linear approximation of GD, for one ('elementary') motionless point particle with mass M in the origin of coordinates ($\mathbf{r}_a \equiv \mathbf{r}_M = 0$ and $\mathrm{v} = 0$) this tensor has the simple form

$$T^{ik} = Mc^2\delta(\mathbf{r})\ \mathrm{diag}(1,0,0,0).$$

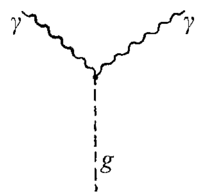
For that massive gravitating centre it fixes the reference frame in which we may investigate the field of such a source.

In the same linear approximation if we base on the interaction $f\,\psi_{ik}\,T^{ik} = f\mu c(\mathrm{d}s/\mathrm{d}t)\ u^i u^k \psi_{ik}$ which one can write down in the symbol form

$f\psi_{ik}T^{ik} \Rightarrow$ 

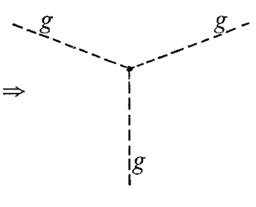
Consistently adhering to the dynamical interpretation of the field ψ_{ik}, we can obtain also the equations of motion for particles in a given field ψ_{ik} just as it is done for an electron moving in a given electromagnetic field (cf. Landau and Lifshitz, 1973).

One can understand as the universality of gravitational interaction in GD the fact that f is *identical* for all fields. In accordance with that, the interaction of electromagnetic field with the gravitational field ψ_{ik} must be written in the form

$f\psi_{ik}t^{ik}_{(el)} \Rightarrow$ 

Ultimately it gives the correct description of interaction effects between light and given gravitational field (such as light deflection and radio signals lag in the field of the Sun). But for the correct description of the redshift effect one must add the interaction $f\,\psi_{ik}\,\theta_{(e)}{}^{ik}$ with the spinor (e^-e^+) field constructed by the same rule, i.e., with the same f to the interaction $f\,\psi_{ik}\,t_{(el)}{}^{ik}$ (Mosinsky, 1950).

In the theoretical scheme based on a consistent dynamical description of gravitational interaction, the introduction of nonlinearities turns out to be directly connected with the field energy problem. The unviersality of gravitational interaction leads to the fact that *any field* (including the gravitational one) interacts with any gravitational field the stronger the higher is its energy. It is evident that the *nonlinear* GD is the interaction when the distance between point particles with mass m_a becomes of the order of Gm_a/c^2. For the collapsar it means that the distance from its centre can reach the value of the order of GM/c^2. I.e., for the field in vacuum the condition $\theta_{00} \sim Mc^2/r^3$ is already satisfied. In accordance with the universality of the gravitational interaction one can assume the field itself to be the source of gravitation (that lacks in ED). It means that in Lagrangian, besides the term $f\,\psi_{ik}\,T^{ik}$, terms arise of the type

$f\psi_{ik}\theta^{ik} \Rightarrow$ 

Here it is seen especially well that the gravitation field energy localizability, as well as the 'pointness' of the particles interacting with the field, can be connected simply with the demand of *locality* of gravitational interaction. (Such a method of introduction of nonlinearities in GD is analogous in a sense to the transition from the **tree** approximation to the one-loop approximation in QED.) Accordingly, in the right-hand side of the field equations it leads to the including of the gravitational field EMT θ_{ik} into the sources also. Then near the collapsar gravitational radius a kind of 'splitting' of the point source $T^{ik} = Mc^2\delta(r)\cdot\mathrm{diag}(1, 0, 0, 0)$ occurs. I.e., in such a case we can say that the degeneration by M is as if it was taken away. A part of the collapsar mass can be now a field mass. It is natural that the accounting of the θ_{ik} in vacuum around the region filled by matter influences also the form of the 4-potential ψ_{ik} of the collapsar field. That leads in particular to a complete explication of observational effects of minor planets perihelions shift, the shift of periastron in a binary system with the radio pulsar PSR 1913 + 16, etc.

The choice of the EMT θ_{ik} of the collapsar strong field is a part of the solution of the collapsar problem in GD. I want to emphasize here that I do not try to describe at once the nonstationary process of collapse as the passage of the system into a bound state, i.e., into the collapsar. It is more simply to state first the problem of probability itself of steady stationary states of the system 'particles + gravitational field' = the system with the given rest energy Mc^2. I.e., the question may be on a possibility (depending on the nearness of an object to its gravitational radius) to speak about such a system as about a 'point' with a definite rest mass M and with the Newton gravitational field (in virtue of the accordance principle) $\varphi_N = - GM/r$ at $r >> GM/c^2$.

Thus we can assume that the problems of the collapsar (but not collapse) in the GD may be: the problem of probability of existence of a *stationary* steady state with a given total energy (Mc^2), the problem of the region dimension (a 'bag' filled by particles-matter) ~ GM/c^2, the problem of the total mass of the 'bag' and the particles density in it, the problem of the field energy density in vacuum around the 'bag' and the total 'mass' contained in the field surrounding the 'bag'.

After this section's remarks we can pass to the substantiation of the choice of the collapsar field EMT. But before, it is necessary at first to elucidate the physical sense of Hilbert-Lorentz gauge condition for the 4-potentials ψ_{ik} in connection with particularities of the interacting gravitational field in vacuum near the 'bag' surface (Sokolov, 1992; hereafter referred to as [PI]).

2. The Components of Vector and Tensor Massless Fields and Gauge Conditions

Certainly, the main purpose of this section is a refinement of the gauge condition sense for the case of the symmetric tensor field ψ_{ik} describing gravitational interaction. But first I shall try to introduce well-known examples from ED whose ideas underline all modern interaction theories.

2.1. VECTOR FIELD

In ED the field Lagrangian is constructed usually from three invariants:

$$I_1 = A_{i,k}A^{i,k}\ , \qquad I_2 = A_{i,k}A^{k,i}\ , \qquad I_3 = A_i{}^{;i}A_k{}^{;k}\ .$$

Since the two last ones differ from each other only by divergence then ultimately for the vector field A_i, interacting with its sources, the Lagrangian consistent only with the condition of relativistic invariance will be in the most general form: i.e.,

$$L_{el} = ½\,(A_{i,k}A^{i,k} - dA_k{}^{;k}A_i{}^{;i}) + j_k A^k \quad . \tag{1}$$

Where d is an arbitrary (auxiliary) constant for the present. Corresponding field equations will be

$$\Box A_k + dA_i{}^{;i}{}_k = -j_k\ , \tag{2}$$

$$\Box \equiv -\frac{\partial^2}{\partial x_i \partial x^i} = \Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}$$

In such a theory there is a 'superfluous' scalar: i.e., the vector A_i is a part of field equations (2) both *directly* ($\Box A_k$) and in the form of a scalar – the 4-divergence $A_i{}^{;i}$ ($dA_i{}^{;i}{}_k$). Corresponding to that, they say that the vector field can also describe simultaneously the scalar field - the scalar component of the field. But if equations (2) describe the electromagnetic field then the experiment demands the use of the a conserved electromagnetic current in (2): i.e.,

$$j_k{}^{;k} = 0\ , \tag{3}$$

Thus, the scalar component is absent in the very source j_k of the field A_k.

In the special case, when d = 1, the whole theory becomes also a gauge invariant one: namely

$$A_i \rightarrow A_i + \Lambda_{,i} \quad , \tag{4}$$

where Λ is an arbitrary 4-scalar. This case (d = 1) combines naturally the absence of the scalar source (3) and the absence of the scalar component ($A_k{}^{;k}$) in the field A_i . The point is that in the gauge invariant theory the very absence of the scalar (3) at d = 1 becomes a *compulsory* (obligatory) condition for the vector source j_i. Such a condition follows the so-called strong law - Noether identity (see, for example, the book by Konoplyeva and Popov (1973) on the second Noether theorem). The strong conservation law ($j_k{}^{;k}$ = 0) is a direct consequence of gauge invariance (4) of the Lagrangian (1) at d = 1. Conformably they say sometimes that condition (3) is necessary for the consistency of field equations with sources (2) at d = 1.

In short, the gauge invariance (4) *demands* the fulfillment of conservation (3). It is well seen in the vector field case in question. Firstly, if we take the 4-divergence of the left-hand side of equations (2) at d = 1, we obtain the identity (Noether identity) in the form

$$\Box A_k{}^{;k} + A_i{}^{;ik}{}_k \equiv \Box A_k{}^{;k} - \Box A_k{}^{;k} \equiv 0\ .$$

This identity (at d = 1) is fulfilled independently of value of the scalar $A_i{}^{;i}$, and this identity demands also the absence of the scalar source $j^i{}_{,i}$ = 0. (Although in particular, it is permissible, that $\Box A^i{}_{,i} \neq 0$.) Second, in accordance with the fact that $j^i{}_{,i}$ = 0, one may use *directly* (at d = 1) the gauge invariance of equations (2) and exclude the superfluous scalar $A^i{}_{,i}$ stipulating for it the gauge condition (Lorentz gauge):

$$A^i{}_{,i} \equiv 0 \quad . \tag{5}$$

If there is no scalar source then it is natural to assume that there is no corresponding field, i.e., the A^i field has no scalar component ($j^i{}_{,i} = 0 \;\Rightarrow\; A^i{}_{,i} = 0$).

In the case of the gauge invariance absence ($d \neq 1$) the excluding of the 'superfluous' scalar $A^i{}_{,i}$ does not appear so natural as at $d = 1$. If we take the divergence of the left-hand side of (2) then it does not yield already the identical zero:

$$\Box A^k{}_{,k} - d\Box A^i{}_{,i} = (1 - d)\Box A^i{}_{,i} \neq 0 \quad .$$

Now, in general, the scalar source may be nonzero, the field equations themselves *do not demand* directly its absence.

But if to demand nevertheless once more (usually they refer here to experiment) the fulfilment of *differential* conservation

$$j^i{}_{,i} = 0 \quad \text{but at} \;\; d \neq 1 \;, \tag{3'}$$

then the equality of divergence in the left-hand side of (2) to zero is ensured as a consequence of the additional demand (3') (that does not follow the theory directly):

$$(1 - d)\Box A^i{}_{,i} = -j^i{}_{,i} \rightarrow 0 \quad .$$

We can say that the scalar part (as the component A_i) is the solution of the equation with the scalar source tending to zero. It is usually said also that scalar photons remain in the theory but they cannot be radiated and absorbed. I.e., the equation $\Box A^i{}_{,i} = 0$ does not mean, generally speaking, that $A^i{}_{,i} \equiv 0$. In that case the demand that $A^i{}_{,i} = 0$ for consistency with (3') means the fulfillment of *one more additional* condition lacking in the theory.

But, on the other hand, ultimately they appeal to the scalar field component A_i requiring the additional degrees of freedom for virtual photons. And what if in this case also we try to adhere to the idea that the account for additional field component is connected with the violation of *differential* law (3') while rigorously observing the *integral* charge conservation?

Then if in the gauge invariant theory ($d = 1$) the conservation $j^i{}_{,i} = 0$ is fulfilled and the field scalar component $A^i{}_{,i}$ (virtual quanta) can be *naturally* excluded from the theory, the case $d \neq 1$ may be considered in a sense as one corresponding to a 'violated' gauge symmetry. In that case it is logical to adopt the fact that the differential law $j^i{}_{,i} = 0$ is not fulfilled already (for almost real and virtual quanta):

$$j^i{}_{,i} \neq 0 \quad \text{at} \quad d \neq 1 \quad . \tag{6}$$

Ultimately, as we could see, there is no change by such point of view in ED but now the arising of scalars in the theory is a more natural alternative. Then the appearance of the scalar is connected with its corresponding scalar source.

In particular, at the quantization of the electromagnetic field they use, namely, the gauge *noninvariant* form of the Lagrangian at $d \neq 1$. Formally it corresponds to the presence of the scalar $A^i_{,i} \neq 0$ in the theory, because of the fact that the condition $A^i_{,i} = 0$ remains fulfilled only *on the average.* Then as far as we may assume that $j^i_{,i} = 0$ also only on the average, the corresponding scalar quanta cannot exist far out of the region of averaging. Thus, if we assume nevertheless the violation of the differential law $j^i_{,i} = 0$ in the region of averaging, i.e., in a sufficiently small (< 10^{-10} cm) space region, then it is in accordance with the absence of gauge invariance in these regions and the presence of the scalar $A^i_{,i}$ is appropriate in them. Indeed, since we may demand the fulfilment of the condition $A^i_{,i} = 0$ only on the average (Bogolyubov and Shirkov, 1973) - i.e., only for average values of

$$\left\langle A^i_{,i} \right\rangle = \Phi^* A^i_{,i} \Phi = 0 ,$$

then we could not assume that the current conservation is fulfilled also, generally speaking, only on the average and in the same permissible states Φ

$$\left\langle J^i_{,i} \right\rangle = \Phi^* j^i_{,i} \Phi = 0 \ .$$

But it can be understood as *differentially* the charge does not conserve near the electron.

Because the analogous reasoning will be used below in GD, now we need a more detailed explanation though the question is here only on some different point of view to *well-known facts*.

At distances less or of the order of $\hbar/m_e c$ (3.6×10^{-11} cm) near the electron the effects of vacuum polarization become more and more important in ED; $e^- e^+$ pairs arise. For all that, the total charge of electron at distances >> $\hbar/m_e c$ is equal to e and is rigorously conserved in accordance with experiment. But also in accordance with experiment this charge is not conserved (differentially) in regions of dimensions < $\hbar/m_e c$. Usually it means an increase of electromagnetic coupling constant $\alpha = e^2/4\pi\hbar c$ at small distances from electron. Such interpretation of $e^- e^+$ pairs influence allows thinking that it is here, where the scalar source differs from zero $j^i_{,i} \neq 0$, the scalar field-scalar photons (virtual photons) must arise.

Thus, generally speaking, the vector field A_i corresponds to particles with two spins: 0 and 1,

$$[\, A_i \,] = 0 \oplus 1 \ . \tag{7}$$

Accordingly, the current j_i can in the general case be a source of particles of two spins:

$$[\ j_i \ ] = 0 \oplus 1 \ . \tag{8}$$

For all this, the scalar parts $A^i_{,i}$ and $j^i_{,i}$ correspond to particles with spin 0. Usually their exclusion in the gauge invariant ($d = 1$) limit with the use of corresponding gauge condition $A^i_{,i} = 0$ and at conservation of current $j^i_{,i} = 0$ remains in the theory only photons with spin 1 – the 'purely' vector for real photons. But as we have just seen, near an electron the photons may have any *possible* spins (0

$\oplus$ 1) which the vector A_i contains. (For example, at count of the scattering amplitude one must take into account *four* possible states of polarization for this set of spins.) As we show in the following section, the analogous situation arises also in GD for field near the surface of the 'bag', i.e., in the strong gravitational field of the collapsar. But here (in the macroscopic theory) the processes of field self-action play the role of quantum processes of pair production.

2.2. SYMMETRIC TENSOR FIELD

Let us pass now to a more complicated case of the symmetric tensor field ψ_{ik}. Here also one can construct in the general case the Lagrangian (following only relativistic invariance for the present) of five quadratic invariants, formed from derivatives of ψ_{ik}:

$$I_1 = \psi_{ik,l}\,\psi^{ik,l}\ ,\ I_2 = \psi_{ik,l}\,\psi^{il,k}\ ,\ \ I_3 = \psi_{,k}\,\psi^{,k}\ ,$$

$$I_4 = \psi_{,l}\,\psi^{kl}{}_{,k}\ ,\qquad I_5 = \psi^{il}{}_{,i}\,\psi^{k}{}_{l,k}\ .$$

As far as I_2 and I_5 are dependent, the requirement of only Lorentz invariance leads to the Lagrangian of the interacting field ψ_{ik} in the form

$$\sum_{A=1}^{4} C_A I_A + \frac{f}{c^2}\psi_{ik}T^{ik}\ , \tag{9}$$

with four arbitrary coefficients C_A, instead of one, whith was in the case of vector field.

If to proceed at once, as for *the main case*, from a self-consistent, gauge-invariant scheme, which is fulfilled in the case of the *linear* GD [P1], then these four coefficient C_1, C_2, C_3, and C_4 are defined straight away as we require invariance of the field Lagrangian and field equations with respect to the gauge transformation of type (4):

$$A_i \rightarrow A_i + \Lambda_{,i} \ \Rightarrow\ \psi_{ik} \rightarrow \psi_{ik} + \Lambda_{,ik}\ , \tag{10'}$$

with an arbitrary 4-scalar Λ. But in that case the symmetric tensor field $\psi_{ik} = \psi_{ki}$ allows more general gauge transformation with an arbitrary *vector field* A_i in the form

$$\psi_{ik} \rightarrow \psi_{ik} + A_{i,k} + A_{k,i} + \Lambda_{,ik}\ . \tag{10}$$

Corresponding relativistic and gauge invariant field equations will be ('long equations')

$$\Box\,\psi_{ik} + \psi_{km}{}^{,m}{}_{i} + \psi_{im}{}^{,m}{}_{k} - \psi_{,ik} - \eta_{ik}(\psi_{mn}{}^{,mn} - \psi^{,n}{}_{n}) = -\frac{f}{2ac^2}T_{ik}\ . \tag{11}$$

Notations are here identical to those in [P1] and T_{ik} denotes as usual the energy-momentum-tensor (EMT) of *point* particles.

Equations (11) lead automatically to a requirement of the strong conservation law

$$T^{ik}{}_{,k} = 0\ , \tag{12}$$

that must, as in ED, correspond in certain situations to the absence of a vector source now already. It is natural (just as in ED for $d = 1$) to assume that if there is no vector source $T^{ik}{}_{,k}$, then by the *direct* use of (10) one may also exclude, in particular, the *vector field* corresponding to this source and contained in the tensor ψ_{ik}.

But here we should understand *what* the vector field corresponds to the vector $T^{ik}{}_{,k}$? If we took the divergence of the left-hand side of equation (11) then it would be identical zero (the Noether identity, or the strong law), which is a consequence of the immediate assumption of the gauge invariance. On the other hand, I shall be interested here mainly in the 'violations' of gauge symmetry which is possible in some situations, for example, in static field of the collapsar, analogously to what was just described for the electron case. I.e., I need a nonzero vector B^i (in a generally speaking gauge *non-invariant* theory) disappearing at the 'restoration' of gauge symmetry and disappearance of a corresponding vector source. In other words, I need the vector B^i consistent with the an equation of type $\Box B^i = -f\, T^{ik}{}_{,k}$.

The point is that the symmetric tensor ψ_{ik} yields a more wide choice of 'superfluous' field components than the vector field A^i. Here it is easy to pick out a scalar $\psi \equiv \psi^m{}_m$, two possible vectors $\psi^{ik}{}_{,k}$ and $\psi^{,k}$ and also two scalars $\psi^{ik}{}_{,ik}$ and $\psi^{,k}{}_{,k}$. One can separate tensor from scalar by invariant manner, using the identity

$$\psi_{ik} \equiv \Phi_{ik} + \tfrac{1}{4}\, \eta_{ik}\psi\,, \quad \text{where } \Phi_i^{\ i} \equiv 0\,.$$

The vector B^i of our interest is a combination of two possible vectors $\psi^{ik}{}_{,k}$ and $\psi^{,i}$:

$$B^i = \psi^{ik}{}_{,k} + b\psi^{,i} = \psi^{ik}{}_{,k} + b\eta^{ik}\psi_{,k}$$

consistent with an equation of type

$$\Box\,(\,\psi^{ik}{}_{,k} + b\eta^{ik}\psi_{,k}) = -fT^{ik}{}_{,k}\ . \tag{14'}$$

In the gauge invariant limit the vector source disappears, $T^{ik}{}_{,k} = 0$ and at this limit the equality $B^i = 0$ can be satisfied. I.e., in that case one can *immediately* use the gauge invariance by a requirement of disappearance of the vector B^i, then the correspondent choice $b = -1/2$ will be determined unambiguously. Here I have only said in other words what is said usually at the Hilbert-Lorentz gauge condition choice: i.e., the foregoing means that the gauge is chosen in such a way that if in the theory just this vector is absent,

$$B^i = \psi^{im}{}_{,m} - \tfrac{1}{2}\,\psi^{,i} = 0 \quad , \tag{13}$$

then 'long' equations (11) are transformed to the known form

$$\Box(\psi^{ik} - \frac{1}{2}\eta^{ik}\psi) = -\frac{f}{2ac^2}T^{ik}\,. \tag{14}$$

It points that condition (13) for the vector B' (in gauge invariant limit) keeps the correctness of what we had in the case of the 'long' equations: identical zeroing of the left-hand side divergence (the Noether identity). Now it is fulfilled also for (14) at $B^i = 0$. One can say that Hilbert-Lorentz gauge (13) just as condition (12) for sources are also the consequence of the strong conservation law in the case of the tensor field ψ_{ik}. But the main thing is here that condition (13) *guarantees* the excluding of the vector field from the theory in the gauge invariant limit and thus the absence of the vector source (12) is consistent with the absence of its corresponding vector field

$$B^i \equiv \psi^{im}{}_{,m} - ½\, \psi^{,i} \equiv \Phi^{im}{}_{,m} - ¼\, \psi^{,i} \ . \tag{15}$$

Just as in the case of the vector field A_i, the symmetric tensor of the second rank ψ_{ik} can be decomposed into corresponding spin parts. Ten independent components of ψ^{ik} can be grouped into two fields of zero spin ($0\oplus 0$), one field of spin 1 ($\oplus 1$) and one of spin 2 ($\oplus 2$):

$$[\psi_{ik}] \Rightarrow 0\oplus 0\oplus 1\oplus 2 \ . \tag{16}$$

For all that, the spin parts, corresponding to 'superfluous' scalar and vector components of the tensor field ψ_{ik}, could be presented in the form

$$[\psi = \psi^m{}_m] \Rightarrow \ \oplus 0 \quad \text{and} \quad [B^i] \Rightarrow \ 0 \oplus 1 \ .$$

And in general case, the symmetrical tensor T^{ik} itself is the source of these fields with four spin parts also – i.e., ten independent components of T_{ik} can be grouped into four sources:

$$[T_{ik}] \Rightarrow \ 0 \oplus 0 \oplus 1 \oplus 2 \ . \tag{17}$$

But in the gauge invariant limit (in the linear GD) the condition $B^i = 0$ at the conservation of the current $T^{ik}{}_{,k}$ retains in the theory the gravitons with two spins. Accordingly, for real gravitons

$$[\psi_{ik}] \Rightarrow \ 0 \oplus 2 \,, \qquad [T_{ik}] \Rightarrow \ 0 \oplus 2 \ . \tag{18}$$

Where the source of purely scalar gravitons is a nonzero trace of the point particles EMT is

$$(T = T^m{}_m) \ \Leftrightarrow \ (\psi = \psi^m{}_m) \ .$$

Thus, one 'superfluous' component (ψ) is, however, possible in the GD (see in detail in [P1]), i.e., in GD there is, in principle, a possibility of real zero-spin gravitons or massless scalar bosons emission.

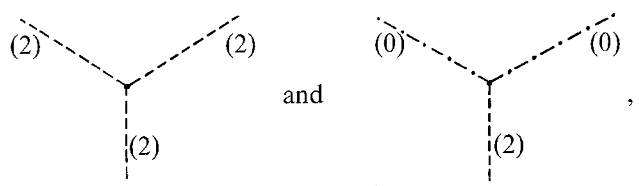
3. Virtual Vector Field of the Collapsar

Here as the initial notion of the collapsar (an 'elementary' object in GD) I shall mean the following: this is a *bound* spherically-symmetric object (an analogue of Schwarzschild's black hole) – a 'bag' filled as before by point particles with $m^*_a \neq 0$ and by fields together with the gravitation field surrounding this bag on the outside. Inside the bag the field cannot be only a gravitational field, it depends on the particles density in the bag. The condition of spherical symmetry is connected with the fact that the 'elementary' object - massive point - yields by definition a spherically-symmetric field around it. It is natural to study first the case of such symmetry of the 'elementary' object down to its centre. Unlike free (or almost free, as in the linear GD) particles with the rest mass m_a we use as in [P1] the designation $m^*_a \neq m_a$ for particles bound inside the bag.

The bound particles with masses $m^*_a \neq 0$ move in this bag in such a way that it is natural to imagine the bag itself and the field around as a certain *stationary state* in which there is a continuous *exchange* between the bag and its surrounding field, *plus* gravitational field self-action processes. The latter are the most essential at small and extremely small dimension of the bag. For all this, if energy of the whole configuration (the bag + field = the collapsar) is constant and equal to Mc^2, then at a constant energy contained in the bag such an exchange leads ultimately to the reaching of the steady-state values of energy-momentum-tensions outside the bag also, i.e., in vacuum surrounding the bag.

For macroscopic objects (the collapsars) with mass M of the order of stellar mass or more the above-mentioned exchange means the existence of a *static* spherically-symmetric (at spherically-symmetric distribution of matter in the bag) gravitational field in vacuum around the bag. As in [P1] I shall speak here mainly about this *external* static vacuum solution of field equations. As to the bag itself, it is for the present sufficient to suppose its spherical symmetry of the distribution and the motion of point particles with the rest masses $m^*_a \neq 0$ bound in the bag (the more specification of the bag features will be in the next paper).

Here the question is mainly on the collapsar gravitational field EMT, i.e., on the choice of an expression for the EMT when the field *interacts* with its sources. Generally speaking, we must now keep in mind also the processes of the field self-action which can be pictured as



(19)

for tensor (spin 2) and scalar (spin 0) gravitons separately. Such processes become determining at a large density θ_{00} of gravitational field energy at distances from the collapsar centre of the order of GM/c^2. In the next section I shall try to elucidate how the field energy is computed *near the bag* with the dimension of order of the collapsar gravitational radius GM/c^2. The designations are here mainly the same as in [P1] though sometimes some more accurate definitions are needed.

Since the collapsar field interacts continuously with the bag (and the bag with the field) and selfacts near the bag in accordance with (19), it is not surprisingly that EMT of this field is not conserved (strictly speaking), i.e.,

$$\theta^{ik}{}_{,k} \neq 0 \quad . \tag{20}$$

On account of the spherical symmetry and static (*stationary*) character of the field under consideration in vacuum, there remain only three identical nonzero components of the 4-vector (20):

$$\theta^{00}{}_{,0} = 0\,, \quad \theta^{11}{}_{,1} = \theta^{22}{}_{,2} = \theta^{33}{}_{,3} \neq 0\,, \tag{20'}$$

(see formula (48) in [P1]). Formally these components can be identified with the presence of a pressure gradient for an 'environment' around the bag. (Of course, the equalities (20') is fulfilled here only approximately, at the characteristic times $>> GM/c^3$. This will be said in more detail in the next paper.)

The nonzero vector $\theta^{ik}{}_{,k}$ must be the source of the corresponding vector field. In general there is no innovation in it, we should only take the 4-divergence of both left-hand and right-hand sides of *vacuum* equation (49) in [P1]:

$$\frac{1}{r}\frac{d^2}{dr^2}[r\delta\Phi_{ik}^{;k}(r)] = -\frac{f}{2ac^2}\theta_{ik}^{;k} \quad , \tag{21}$$

where the equation is written only for the addition $\delta\Phi_{ik}$ to the potential $\Phi_{ik}{}^{(p)}$ of the 'point' source in [P1] for $\Phi_{ik} \equiv \Phi_{ik}{}^{(p)} + \delta\Phi_{ik}$ (see formula (28') in [P1]). Here only the new designation $\Phi_{ik}{}^{(p)} \equiv \Phi_{(p)}{}^{ik}$ is introduced here for the 4-potential (28) in [P1]:

$$\Phi_{ik}^{(p)} \equiv \frac{3}{2}\frac{f}{16\pi a}\frac{M}{r}diag(1,\frac{1}{3},\frac{1}{3},\frac{1}{3}) \ .$$

It is obtained in [P1] without account for θ_{ik} in the right-hand side of the field equations, i.e., at $r >> GM/c^2$. For the structureless 'point' source the Hilbert-Lorentz gauge condition $\Phi_{(p)}{}^{ik}{}_{,k} = ¼\, \psi^{,i}$ is still satisfied at this approximation. I.e., the relevant vector field is absent, which means that

$$\Phi_{(p)}{}^{ik}{}_{,k} - ¼\, \psi^{,i} = 0\,. \tag{22}$$

Certainly, at more logical reasoning one should keep in mind that equation (21) is a consequence of the more general equation

$$\square(\psi^{ik} - \frac{1}{2}\eta^{ik}\psi)_{,k} = -\frac{f}{2ac^2}(T_{(*)}^{ik} + \theta^{ik})_{,k} \tag{23}$$

with the *nonzero* (differentially) right-hand side already. This equation must be written down excluding the bag (i.e., the region with $T^{ik}{}_{(*)} \neq 0$ for bound particles) and taking into account the spherical symmetry

$$\frac{1}{r}\frac{d^2}{dr^2}[r(\Phi^{ik}-\frac{1}{4}\psi\eta^{ik})_{,k}]=-\frac{f}{2ac^2}\theta^{ik}_{,k} \quad .$$

Now condition (22) is not satisfied already for the vector field $B^i = \Phi^{ik}_{,k} - \frac{1}{4}\psi^{,i}$. I.e., the Hilbert-Lorentz gauge condition for the 4-potential $\Phi^{ik} = \Phi_{(p)}^{ik} + \delta\Phi^{ik}$ (see formula (28') in [P1]) obtained with the account of field self-action processes (19) *is not satisfied.*

By use of the fact that gauge condition (22) is satisfied for the 'point' potential $\Phi_{(p)}^{ik}$ we shall really have for the left-hand side of (21):

$$\frac{1}{r}\frac{d^2}{dr^2}[r(\Phi^{ik}_{(p)}+\delta\Phi^{ik}-\frac{1}{4}\psi\eta^{ik})_{,k}]=\frac{1}{r}\frac{d^2}{dr^2}[r\delta\Phi^{ik}_{,k}] \quad .$$

Thus the vector B^i which arises in the nonlinear approximation of GD or at the refusal of the gauge invariance is equal to

$$B^i = \Phi^{ik}_{,k} - \tfrac{1}{4}\psi^{,i} = \delta\Phi^{ik}_{,k} \neq 0 \quad , \tag{24}$$

i.e., this vector is reduced to vector 4-divergence of nonlinear addition $\delta\Phi^{ik}$ to 'point' potential $\Phi_{(p)}^{ik}$. The vector B^i arises here as a consequence the very gravitational field as the sources, in other words, because of the accounting (next approximation in the Lagrangian of interaction (9)) for the process of type (19). Of course, for all that the gauge invariance is already violated.

I want to emphasize once more that there is no absolutely new vector field here. If we assume that near the bag (at $r \sim GM/c^2$) the differential conservation is *not fulfilled* for sources (in this connection it is appropriate to recall differential and integral conservation in GR):

$$T^{ik}_{(*),k} + \theta^{ik}_{,k} \neq 0 \quad \text{at} \quad r \simeq GM/c^2 \quad , \tag{25}$$

then the vector component of the tensor field which was a purely tensor component at $r >> GM/c^2$, is sure 'to start operating' (see [P1]). (For all that (20) is a particular case of (25) for vacuum.) This vector component was excluded then because of an absence of the appreciable input of the vector source at $r >> GM/c^2$. But at $r \sim GM/c^2$ *everything* that the symmetrical tensor of the second rank (16) yields *is used.*

Thus from the foregoing the conclusion is that near the bag (at $r \sim GM/c^2$) the tensor component Φ_{ik} of the collapsar field ψ_{ik} gives raise an additional vector component. For all that we can say that an additional degree of freedom is 'unfrozen' - the vector component Φ_{ik} of the field (and after it the scalar $\Phi^{ik}_{,ik} \neq 0$ also). Now we may not say about Φ_{ik} as about a purely tensor field, we may say about tensor field without one scalar. I.e., the field Φ_{ik} contains at $r \sim GM/c^2$ three spin parts: i.e.,

$$[\Phi_{ik}] \Rightarrow 0\oplus1\oplus2 \ , \quad \Phi^m_{\ m} \equiv 0 \quad . \tag{26}$$

What occurs with tensor field Φ_{ik} at $r \to GM/c^2$ is natural from the quantum-field point of view. The nearer to the bag the more virtual this field becomes. Accordingly (and analogously to what was in QED), in that case the components of spin 1 (and 0) must appear, lacking in real gravitons of spin 2. But at 'unfreezing' of additional components of the field Φ_{ik} the question arises on correct account for contribution of these components into the field energy.

In case of the purely scalar field $\psi = \psi^m_m$ there is nothing to 'unfreeze'. There remains also only one component even near the bag. For this field (with always well-determined spin) both real and virtual gravitons have spin 0. The last is in good agreement with the fact that energy dependence on r for the purely scalar field remains invariable up to the distance of the order of $\sim GM/c^2$ from the bag centre. (Of course, it is fulfilled till one may speak about gravitation only, see [P1].) For the collapsar massless gravitational field the EMT of the ψ-component at any distance from the bag centre up to $r \approx GM/c^2$ is equal [P1] to

$$\theta^{ik}_{(0)} = \frac{1}{16\pi}\frac{GM^2}{r^4} diag(1,\frac{1}{3},\frac{1}{3},\frac{1}{3}) \quad . \tag{27}$$

It is this fact which allows me insisting confidently in [PI] on absence of the singularity in GD. In what follows, also at computing of the collapsar tensor field energy, I will do my best to use the possibility of division of the field ψ_{ik} into two components Φ_{ik} and ψ, when the scalar component features are clear in many respects.

4. The Energy-Momentum Tensor of the Interacting Gravitational Field of the Collapsar

Now I pass directly to the apparently most complicated moment in the whole scheme of GD. I will try to understand how the EMT of the tensor component Φ_{ik} of the field ψ_{ik} behaves close to the bag surface or at distances from its centre of the order of GM/c^2, i.e., where *all* possible components of the tensor field come into play ($[\Phi_{ik}] \Rightarrow 0 \oplus 1 \oplus 2$), or where tensor gravitons depart from their mass surface and become essentially virtual particles, for example, where they exist only a short time of the order of $(GM/c^2)/c \sim 10^{-4}$-10^{-5} s for stellar mass objects and correspondingly huge ($> 10^{14}$ g cm^{-3}) density of the gravitational field on surface of such bags.

Let us take once more the example of the vector field in ED. In this theory, in principle the same formula for the electromagnetic field energy density may be applied both in the case of a free field (corresponding to real photons) and in the case of a field around of a source (electron), i.e., in the case of a static field in vacuum (corresponding to almost real and virtual photons) interacting with the source. It is possible, at least in principle, because the obtaining of the field EMT in ED is done *without* use of the Lorentz gauge $A^i_{,i} = 0$ (see Landau and Lifshitz, 1973). Consequently, the scalar component of the vector field A^i is not excluded and this additional degree of freedom may, in principle, yield its contribution into the EMT of electromagnetic field.

Of course, here the question is only on possibility in principle of application of identical formulae for the field energy in the classical ED in two situations of 'charge absence'. Strictly speaking, the same formula for energy is applicable both in case of the free field and in the case of the field interacting with its sources only until photons near the charge (electron) depart considerably from their mass surface, i.e., photons must nevertheless be sufficiently 'almost real' for one may neglect the quantum effects of the $e^- e^+$ pair creation.

Coming back to GD in the scheme, suggested in [P1], the obtaining of the field EMT is substantiated only for *every* component ψ and Φ_{ik} of the *free* gravitational field, i.e., of the field in the wave zone as a matter of fact. In this case one manages to secure confidently the positive definiteness of the field energy also. Therefore, strictly speaking, in GD (unlike what is at least in principle permitted in ED) one may not use directly, without reservation, the 'purely tensor' EMT formula (see formula (43) in [P1] for of the field with definite spin 2) when the tensor Φ_{ik} has a vector part also. This can be particularly important in the case of a 'strongly' interacting (self-acting, more exactly here) gravitational field near the bag surface at $r \approx GM/c^2$, when the vector $\theta^{ik}{}_{,k}$ differs considerably from zero: i.e., this is the case of essentially virtual gravitons - the vector B^i is 'unfrozen' at least. But here one should keep in mind that apparently there is no method (like the analogous situation in QED) of Lorentz-covariant division of the field Φ_{ik} into components for such virtual gravitons unlike what was made in [P1] for real gravitons.

I emphasize once more that in [P1] the EMT formula was obtained for the free field real gravitons. Then it was applied (see equation (48) etc. in [P1]) for the case of the collapsar field, i.e., for virtual gravitons. Now it requires elucidations.

Apparently, the first step which was made in [P1], i.e., the substitution

$$\Phi_{(\mathrm{p})}{}^{ik} \rightarrow \theta^{ik}{}_{(2)} \ ,$$

proves its value by the fact that at $r >> GM/c^2$ we still deal with almost real gravitons of spin 2. But the next step like all the other 'approximations'

$$\Phi^{ik} = \Phi_{(\mathrm{p})}{}^{ik} + \delta\Phi^{ik} \rightarrow \theta^{ik}{}_{(2)} \ ,$$

will be of a small sense at the use of the same 'pure' EMT formula $\theta^{ik}{}_{(2)}$ (for gravitons with the *definite* spin 2, see (43) in [P1]) for the tensor component of the field.

An approach to the bag means that gravitons become more and more virtual, they depart more and more from their mass surface ($p^i p_i \neq 0$). One *can assume* that in that case the application of formula (43) from [P1] selects ('cuts out') the energy *only* for the spin 2 particles, though the vector $B^i = \delta\Phi^{ik}{}_{,k}$ is nonzero already and conformably the particles of spin 0 and 1 appear. For all that this part of the total EMT (for spin 2 only) remains diagonal and traceless as before.

If here also, as in the case with virtual photon, there is no a covariant selecting method for additional components of virtual graviton, then correspondingly there is no a general method of the deduction of EMT of such gravitons (as a conserved quantity of a close system). There is a little sense in such 'deduction' because virtual particles are essentially interacting particles. We may speak about EMT of such particles only as about a stationary set in (for example, as a result of an exchange with the bag + the self-action processes) values of energy-momentum-tensions of gravitational 'coat' around the bag.

Of course, one may hope that the total theoretical solution of the problem of energy of interacting gravitons will be at last carried out in a totally quantized GD. Here, like QED, the essential will be the question on a graviton propagator choice in the theoretic scheme under consideration with two types (ψ and Φ_{ik}) of free fields. But it seems to me that *preliminarily* one can understand much in GD by the careful study of the sense of gauge conditions, conservation laws, etc., it is this that is made in this paper. One must always keep in mind that the question is mainly on the macroscopic theory (GD) and macroscopic objects. Remarks on these features which differ GD from microscopic quantum theories, such as QED and QCD, will be made at the end of the paper.

Now I attempt to move forward going by the way of natural physical suppositions and continuing the use of the quantum field theory notions. For all that I shall always mean *macroscopic* situations, when there are so many gravitons that one may speak to (still with a high precision) about some 'environment' around the bag, properties of which are given by the tensor θ_{ik}. Ultimately observable effects predicted in this direction can resolve the gravitons energy problem *by experiment* as it was as a matter of fact in QED.

Below we shall use here the signs $\theta^{ik}{}_{(\psi)}$ and $\theta^{ik}{}_{(\Phi)}$ for EMTs of the ψ and Φ_{ik} components correspondingly for the interacting fields near the bag. And the interacting scalar field EMT coincides with EMT (27) of real gravitons of spin 0: $\theta^{ik}{}_{(\psi)} = \theta^{ik}{}_{(0)}$.

Let us assume that:

1) Since the spherical symmetry remains valid also up to $r \approx GM/c^2$ it is natural to suppose that the 'unfreezing' of additional components of the field tensor Φ_{ik} does not 'spoil' the diagonality of its EMT $\theta^{ik}{}_{(\Phi)}$.

2) If the field Φ_{ik} remains for $r \approx GM/c^2$ also massless as before (and quantum corrections/effects are still small), then such an 'unfreezing' of spins $0 \oplus 1$ must not 'spoil' also the tracelessness of the EMT $\theta^{ik}{}_{(\Phi)}$, taking into account all spins.

3) At last, the assumption remains valid about the energy positivity $\theta^{00}{}_{(\Phi)}$ for the interacting field Φ_{ik} (of integer spin) even at the 'unfreezing' additional spins; and what is more, I continue to consider that the energy of these additional components remains positive (at least in sum) for spins 1 and 0 in Φ_{ik}.

Thus, near the bag for the case of the interacting field Φ_{ik} the EMT $\theta^{ik}{}_{(\Phi)}$, taking account of *all* spins, must remain as before diagonal, traceless and always with a positively determined $\theta^{00}{}_{(\Phi)}$-component.

However, now let us make the substitution

$$\Phi_{ik} = \Phi^{(p)}{}_{ik} + \delta\Phi_{ik} \rightarrow \theta^{ik}{}_{(2)} \quad ,$$

in equation (43) from [P1] for the EMT $\theta^{ik}{}_{(2)}$ which takes into account only spin 2 in Φ_{ik}. As a result we obtain the diagonal tensor

$$\theta^{00}_{(2)}(1-4\frac{r_e}{r}+4\frac{r_e^2}{r^2})diag(1,\frac{1}{3},\frac{1}{3},\frac{1}{3}) \quad , \tag{28}$$

where $\theta^{00}{}_{(2)}$ = $(1/16\pi)(GM^2/r^4)$ and the $r_e \equiv 1/3\ GM/c^2$ is introduced here for the notational convenience. I would remind once more that the question is always on the space regions in vacuum around the bag at $r > GM/c^2$ or $r \approx GM/c^2$, i.e., the bag itself is excluded from consideration.

In comparison with what was at the substitution in the formula for $\theta^{ik}{}_{(2)}$ accounting only for spin 2 in [P1] of the 'point' potential $\Phi^{ik}{}_{(p)}$ (the case of almost real gravitons), we note now *the decrease* of energy for every r by the factor

$$(1-4\frac{r_e}{r}+4\frac{r_e^2}{r^2}) \le 1 \quad \text{at} \quad r \ge r_e \ ,$$

and as a consequence, the condition of the equality of energies in every point of space for both components ψ and Φ_{ik}, which was fulfilled in the case of almost real gravitons (see formula (47) in [P1]) is now broken

$$\theta^{00}{}_{(\psi)} \equiv \theta^{00}{}_{(0)} \ge \theta^{00}{}_{(2)}\ (1-4\frac{r_e}{r}+4\frac{r_e^2}{r^2}) \quad . \tag{29}$$

But now (at $r \approx GM/c^2$) both types of gravitons (ψ and Φ_{ik}) can already be essentially virtual particles. If one considers that the application of formula (43) from [P1] selects ('cuts out') the energy of only spin 2 gravitons in Φ_{ik}, then the decrease of the right-hand side of (29) is explainable: simply in the right-hand side of inequality (29) it is not everything that is accounted for.

Suppose that a contribution of 'unfrozen' additional virtual components of the tensor field Φ_{ik} in accordance with the above-mentioned assumptions 1), 2), 3) is given by the tensor

$$\theta^{00}_{(2)}(4\frac{r_e}{r}-4\frac{r_e^2}{r^2})diag(1,\frac{1}{3},\frac{1}{3},\frac{1}{3}) \quad . \tag{30}$$

Then if we add it to (28) we obtain for EMT of the field Φ_{ik} the 'old' expression

$$\theta^{ik}_{(\Phi)} = \theta^{00}_{(2)} diag(1,\frac{1}{3},\frac{1}{3},\frac{1}{3}) = \theta^{ik}_{(2)} \quad , \tag{31}$$

in which the decrease of energy in (29) is *compensated* by the positive energy of the 'unfrozen' virtual components from diagonal and traceless tensor (30).

As a matter of fact, a statement is formulated here which may serve as a basic, *experimentally* *verifiable assumption* for the gravitational field around the bag: energy of the tensor field Φ_{ik} with all its possible spin parts ($[\Phi_{ik}] \Rightarrow 0 \oplus 1 \oplus 2$) is equal as before to the energy of purely scalar field ψ in every point, i.e. at *any* $r > GM/c^2$ or $r \approx GM/c^2$. It means that in every point around the bag the condition

$$\theta^{ik}{}_{(\psi)} = \theta^{ik}{}_{(\Phi)} \tag{*}$$

is fulfilled (see condition (47) in [P1]) which is true also for virtual gravitons of the collapsar static or stationary (more exactly) field.

Thus equation (27) ($\theta^{ik}{}_{(0)} \equiv \theta^{ik}{}_{(\psi)}$) together with the condition (*) are proposed here as a solution of the energy problem for the interacting collapsar field, if a bag radius $R \geq GM/c^2$ and M of order of solar masses and greater. From the foregoing it follows that the energy-momentum-tension for the 'environment' around the bag of such a (macroscopic) dimension can be as before given by the tensor

$$\theta^{ik} = \theta^{ik}_{(\psi)} + \theta^{ik}_{(\Phi)} = \theta^{ik}_{(0)} + \theta^{ik}_{(2)} = \frac{1}{8\pi}\frac{GM^2}{r^4} diag\left(1, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right) \quad . \tag{32}$$

In that case the tensor potential Φ_{ik} for the outer field of the bag will have in *all* 'approximations' the identical appearance, coinciding with formula (28') in [P1]. I.e., nonzero components of the potential ψ_{ik} for the collapsar field in vacuum will be, as before,

$$\psi_{00} = \frac{GM/f}{r}\left(1 - \frac{1}{2}\frac{GM/c^2}{r}\right) \quad , \tag{33}$$

$$\psi_{11} = \psi_{22} = \psi_{33} = \frac{GM/f}{r}\left(1 - \frac{1}{6}\frac{GM/c^2}{r}\right) \quad .$$

5. Conclusions

It should be said that the theoretical scheme developed in [P1] and in this paper differs radically from different variants of theoretical alternatives to GR by the fact that here gravitations are two types: scalar and tensor ones interacting with the matter by the identical coupling constant. I.e., there is a scalar field ψ – the unremovable 'superfluous' field component which corresponds to the scalar source T. Thus we develop the variant of the theory in which side by side with real spin 2 massless gravitons there may exist real massless particles of spin 0 – massless scalar bosons.

Unlike what was made in the paper by Sexl (1967), our scheme contains massless scalar field which is included in still *completely* gauge invariant theory with 5 gauge functions (10). What was called in the paper by Sexl the expulsion of the scalar ψ, in our case is only the *separation* of the scalar part from the tensor one in free field with 5 gauge conditions for the tensor component (see in [P1]).

Furthermore, it is essential to distinguish between two approximations in the theory:

(1) *The linear GD* is a gauge-invariant theory where sources may be only points or a system of massive points bound by massless fields (gauge fields) as it is usual in a local theory of field. It is in this approximation where the Lorentz covariant division is made into scalar and purely tensor gravitational fields. The law $T^{ik}{}_{,k} = 0$ means first of all the mass conservation law for interacting particles.

(2) *The nonlinear GD* is a gauge-noninvariant theory. I.e., the introduction of non-linearities into GD is connected first and foremost with the 'violation' of the gauge invariance. In this approximation the mass of interacting particles is not conserved; the law $T^{ik}{}_{,k} = 0$ (in connection with the violation of the gauge symmetry (10)) is not already fulfilled in the differential sense.

In the nonlinear approximation of GD the vector divergence of the sum $(T^{ik}{}_{(*)} + \theta^{ik})_{,k}$ is not equal to zero (25) in the collapsar strong field (near the bag) when elementary volumes of averaging are less than or of the order of $(GM/c^2)^3$. At distances $r >> GM/c^2$, where the linear approximation of GD is correct, the elementary volumes by which the averaging is made in the differential law, may be much greater than $(GM/c^2)^3$. It results in the transition

$$(T^{ik}{}_{(*)} + \theta^{ik})_{,k} \rightarrow T^{ik}{}_{,k} = 0 \quad ,$$

which can be understood as the conservation law for particles together with their gravitational 'coats' (i.e., at $r >> GM/c^2$ the 'degeneration' by M appears).

We connect the introduction of non-linearities into GD first of all with the accounting in the interaction Lagrangian for the terms of type:

$$f\psi_{ik}\theta^{ik} = f\psi_{ik}\theta^{ik}{}_{(0)} + f\psi_{ik}\theta^{ik}{}_{(2)} = f\Phi_{ik}\theta^{ik}{}_{(0)} + f\Phi_{ik}\theta^{ik}{}_{(2)} \; ,$$

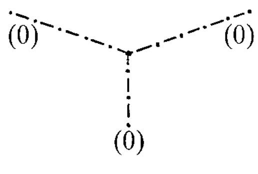
which we can correlate with corresponding elementary processes (19) in accordance with general laws. In a sense (see Section 1) it corresponds to transition from the tree approximation to the one-loop approximation in QED. But there it is true that the terms $f\psi_{ik}\theta^{ik}$ *violate* the gauge invariance (10) of the theory. (There are self-action processes of type (19) in QCD, but there they are included *primordially* in the gauge invariant scheme because of the non-Abelian character of the gauge group.)

In this paper the quantum-field notions are often used and even elementary processes are mentioned that is more appropriate in a totally quantized theory. Indeed, after that I tried in [P1] and in this paper to elucidate the sense of gauge conditions one can try to approach to the procedure of quantization of the proposed variant of theoretical scheme. But in collapsar macroscopic cases under consideration quantum-field and classical notions coexist simultaneously. We speak about virtual and almost real gravitons. For the collapsars with the mass 10 $M_{\odot}$ the virtual gravitons exist as real particles about 10^{-4}s – these are rather big times in comparison with the muon lifetime (e.g.). The almost real gravitons exist more than 3 min in case of the Sun - the Mercury exchange. The virtual gravitons in the case of cosmological objects with dimensions of their gravitational radius of the order of several astronomical units exist already during the hours as real particles.

On the other hand, since for the description of the collapsar properties with the mass ~ M_Θ one may use the tensor θ_{ik} for the same 'environment' around the bag. It means that there are many gravitons in every $(cm)^3$ for the bag around with the radius greater than or of order of several kilometers. First of all here the 'weight' of every such $(cm)^3$ of 'vacuum' around the bag is important. It becomes comparable with the 'weight' of $(cm)^3$ inside the bag itself. Then the accounting for the gravitation of such 'vacuum' becomes important before the caring about quantum effects.

The above remarks have for an object to emphasize the difference between macroscopic theory which GD is, and microscopic ones – the quantum theories QED and QCD with massless gauge fields. But here we emphasize also the common ideological base of GD and these theories. In particular, in the consistent dynamical theory of gravitational interaction one can and must seriously speak about virtual gravitons, not only about real ones. The very problem of quantization of gravitation inevitably poses the question about virtual gravitons (existing 'for a long macroscopic time' for the macroscopic collapsars), the main property of which, like of photons in QED and gluons in QCD is the presence of *all possible polarization states* for the symmetric tensor field (16). One can be occupied with the choice of a graviton propagator in this case. But at least it is clear already that the ('lost' in the linear GD) vector $B^i = \psi^{im}{}_{,m} - ½\ \psi^{,i}$ must be restored in the scheme with all the ensuing consequences for field energy and potential ψ^{ik} near the bag. Before the fulfilment of a total quantization procedure it is questionable whether the equality $\theta^{ik}{}_{(\psi)} = \theta^{ik}{}_{(\Phi)}$ (*), based still on common reasonings, is sufficiently well-grounded. But my purpose in this paper would be half achieved if I managed to convince the reader that the account for all components of the field Φ_{ik} is as necessary in GD (in the case of the strong collapsar field) as in QED and QCD in analogous cases demanding the accounting for all possible spin states of virtual photon and gluon.

Indeed, since the condition $\theta^i{}_i = 0$ causes the linearity of the purely scalar component ψ of gravitation, i.e., there is no vertex



then satisfying once (in gauge invariant scheme still) the condition of the vector absence

$$\psi_{(p)}{}^{ik}{}_{,k} - ½\ \psi^{,i} = \Phi_{(p)}{}^{ik}{}_{,k} - ¼\ \psi^{,i} = 0\ ,$$

one can *never* do that at $r \sim GM/c^2$. The vector field $B^i = \delta\Phi^{ik}{}_{,k} \neq 0$ will counteract. Certainly one may doubt the choice of field EMT (32), but the taking into account of additional spins is necessary nevertheless. Thus the quantum properties of the field ψ_{ik} – the presence of additional spins in virtual particles – become apparent already in the half-classic, macroscopic situation with the collapsar.

The tensor potential of the static field of the bag (33) obtained by allowing for the equality (*) must lead to certain experimental (observable) consequences. It could be testified in particular by the periastron shift effect in relativistic close binary systems. The shift effect must be described by the same 'old' formula [P1] as

$$\delta\varphi = \delta\varphi_1 + \delta\varphi_2 = \frac{6\pi}{(1-e^2)} \frac{GM/c^2}{a}$$

fulfilled because of (*) at all $r >/\sim GM/c^2$. Certainly, for all that one must not forget to take into account the more and more increasing role of gravitational emission from the system influencing the secular effects what is particularly important for small values of $r \sim GM/c^2$.

A special paper will be dedicated to the study of observational consequences for potential (33). Ultimately, from (*) one must obtain the picture of properties of the bag, of its surface and the field around, consistent with all other physics. These properties must lead to absolutely definite experimental (observational) tests allowing distinguishing the collapsars in GD from black holes in GR.

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