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LINEAR AND NONLINEAR GRAVIDYNAMICS: STATIC FIELD OF A COLLAPSAR

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Abstract. In the context of a consistent dynamic interpretation of gravitation (gravidynamics) the gravitational field has been divided into two components: scalar and tensor, each one interacting with its source with the same coupling constant. Consequently, the spherically-symmetrical gravitational field generated by a massive object (a source) influences test bodies as an algebraic sum of attraction and repulsion. The field energy in vacuum around the source is also a sum of energies of two components - purely tensor and scalar ones of gravitation. At distances from a gravitating object much greater than its gravitational radius, energies of each separate field component are equal to each other at the same point of space.

In the bounds of the gravidynamics based on the so-called Einstein's 'linearized' equation and proceeding from general principles of theory of classical fields a statement (a theorem) has been formulated on the static gravitational field of a collapsar: a spherically-symmetric object generating a static field in vacuum can always occupy only a finite, nonzero volume.

1. Introduction

The question we wish to raise below concerns an outer gravitational field generated by a spherically-symmetric distribution of matter in vacuum from the point of view of a consistent dynamic interpretation of gravitation. We have in mind a certain spherically-symmetric configuration of the system 'matter + gravitational field' with any radius of the sphere filled by matter down to the smallest dimensions of the sphere (a compact configuration) of the order of GM/c^2, where G is the gravitational constant and c is speed of light. The total energy of every one of these configurations coincides with the rest energy of the system and is denoted as Mc^2. This paper tries to answer such a concrete question as what are limits where the outer gravitational field under the discussion can still be considered as static.

One can formulate the question in another way: to what extent one can compress matter for an obtained configuration to have still a static field in vacuum? The paper, however, concerns mainly a stationary stable state of the system 'matter + gravitational field' (i.e., a collapsar), but not the very process surely nonstationary of the collapse, i.e., of the compression of matter to dimensions of a sphere with a radius of the order of GM/c^2. The collapse as a process of transition of a system to a more bound state is not considered here. For the present the author's aim is to study the stationary spherically-symmetric configuration with the greatest possible binding energy from the point of view of dynamic field theoretical interpretation of gravitation that we have been adhering to in our papers (Sokolov and Baryshev, 1980; Baryshev and Sokolov, 1984).

2. Energy of Gravitational Field in Gravidynamics and the Static Field of Collapsar

As is generally known, the problem of gravitational field energy exists in the General Relativity (GR) nearly since the moment of its discovery, and the debate (in particular) on localizability and positivity of this energy in GR continues up to the present. More simply, it is not clear till now how to understand the conservation of energy in GR. One of the most known articles on this topic by Zel'dovich and Grishchuk (1986) does not solve, as earlier, all the problems related to field energy. But these authors agree at least that one can try to explain gravitation dynamically, i.e., without identifying it with geometry of space-time by GR. They concluded that, all the same, as a result we shall come to the same GR. But not all physicists share such an opinion (see e.g. Logunov and Mestvirishvily, 1985; Vlasov and Logunov, 1987), in particular, when it concerns the question on the field of a collapsar. This static and spherically-symmetric field in vacuum around a region filled by matter will be discussed below from the point of view of gravidynamics (GD). Thereby, since the very beginning we accept *as axiom* here that the energy of gravitational field is localizable, positive and understood in the same sense as it is understood in the classical electrodynamics (ED).

What is concretely this energy, or rather energy density, for example, in the case of the collapsar static field? In ED, the field energy is defined by the second power of electric and magnetic fields. In GD the gravitational field energy density is also defined by the second power of its field strength g^2/G, where g is the gravitational acceleration. As we have shown in our paper (Sokolov and Baryshev, 1980), the field energy density near a gravitating body can be calculated by the formula

$$\theta_{00} = \frac{1}{8\pi G}(\nabla\varphi_N)^2 \equiv \frac{g^2/G}{8\pi}. \quad (1)$$

Where $\varphi_N = -\dfrac{GM}{r}$ is the usual Newtons's potential. Subsequent sections of this paper will consider in detail the choice of particularly this formula for the static field energy density. In this section it is shown to what consequences for the collapsar field the fulfilment of *natural* requirements of positiveness and localizability of gravitational field energy can lead.

In our opinion, the old debate on the field energy is first of all a debate on our understanding (our 'reading' or interpretation) of equations describing the gravitational field. One of the aims of this paper is to look from another point of view at 'old' conceptions by describing where possible in detail the physical sense of equations, formulae, idealizations known for a long time. I would like to emphasize right now that the base of our point of view on gravitation is the consistent, dynamic, field-theoretic interpretation of the same 'old' Einstein's equations. (But Einstein's equations are written from the point of view of GR in the so-called 'linearized' form.) In GD it concerns both weak and strong fields eventually.

The following shows that the choice of a correct formula for θ_{00} is connected with the fact that the gravitational field energy continuously distributed in vacuum around the Sun directly affects the Mercury perihelion shift. It turns out that for the right explication of such an effect in GD the field energy must be positive only: i.e., the localization of energy (it means that the amount of field energy around the Sun in any volume or in every cubic cm is definite) and its positiveness (it means that field energy is positive like any mass) are directly connected in GD with observations, with experiments. Just this fact (noticed for the first time by Thirring, 1961) sets out the possibility of the dynamic interpretation of gravitation for elucidation of the meaning and the value of θ_{00}.

But θ_{00} is only one, temporal component of the energy-momentum tensor (EMT) θ_{ik} of gravitational field. The choice of the formula (1) for θ_{00} besides the symmetry $\theta_{ik} = \theta_{ki}$ must take into account the fact that the gravitation field is massless field. In other words, the field action radius r is unlimited and the corresponding field quanta (gravitons) are particles with the zero rest mass. All this is available in ED. In ED the field masslessness is connected in particular with the lack of a trace of the electro-magnetic field energy-momentum tensor (EMT). Correspondingly, in the GD for the gravitational field EMT we proceeded from the condition

$$\theta^{ik}\eta_{ik} \equiv 0, \quad (i, k = 0,1,2,3). \qquad (2)$$

Where η_{ik} = diag(+ 1, - 1, - 1, - 1) is the diagonal Minkowsky's metric tensor. Thus, for the following, an assumption that three conditions (or axioms) $\theta_{00} > 0$, $\theta^{ik}\eta_{ik} \equiv 0$, $\theta_{ik} = \theta_{ki}$, which influence the choice of the formula for θ_{00} are satisfied, is of great importance.

Since the present work concerns, as a matter of fact, the features of problem statement and this section will show how to answer questions posed in the Introduction, we are going to elucidate here in detail the sense of the so-called 'point idealization'. Ultimately, the difference between the linear and nonlinear GD, will be clear at once, a gravitational radius will appear as the main parameter of GD.

The fact is that in any classical relativistic field theory (in ED and in GD) one may strictly speaking not ascribe without reserve some finite dimensions both to test particles and to particles which are the sources of field. In other words, only point sources or a system of point sources as a macroscopic gravitating body in GD may be on the right side of corresponding field equations. It is connected with a specific character of Special Relativity (SR) in which all extent bodies are to be represented as a system of point (structureless) objects interacting with each other.

In particular, the 'points' with masses m_a which real macroscopic gravitating bodies consist of ($\sum_a m_a$) are meant to be entire regions of generally macroscopic dimensions with masses m_a. Certainly, these are not molecules, atoms, and electrons. These are large macroscopic regions between which basically gravitation force is acting only. In the following I shall do my best to emphasize and to use this fundamental conception of interacting points in SR (or in the classical field theory) as an initial notion of a 'gravitational charge'. Here (in the GD) a natural question arises: to what limits is this idealized concept useful and acceptable?

In exposition/description of GD as in ED, it is methodologically convenient, at least at the beginning, to eliminate all other interactions but the gravitational one only. As was noted before, the most general form of a field source (a macroscopic gravitating body) is a system of point sources interacting with each other only gravitationally. (But one must think how to provide the stability of such a system – see Introduction.) At first we investigate what is here (in the GD) a single, stationary, or an 'elementary' point source with the mass M and with the static field in vacuum. One may use an analogy with the elementary point charge (electron) in the classical ED.

Thus, every point generates around itself a static spherically-symmetric field. Let this field point source resting (for the reference frame definition) at the origin of coordinates. For the field creation it is necessary to spend some energy, i.e., the field around the point source contains some part of the source mass. It means that if the field energy is sufficiently large we deal with a material object distributed continuously (and spherically-symmetric as before) in space around. In that case in ED they say about electron surrounded by a 'fur-coat' of virtual photons. In ED the field around the point the electron will be characterized by the definite energy density $E^2/8\pi = e^2/8\pi R^4$. When this density becomes comparable with the mean rest energy density $m_e c^2/R^3$ of the particle= electron, the question arises: where is the mass of the electron concentrated (in a volume ~ R^3)? This is the old ED problem, still unresolved completely in its quantum generalization. But as is generally known, the classical (and linear) ED is applicable till the distance between charges is much larger than the classical electron radius $e^2/m_e c^2$. And this is conventionally a 'point' (the point source) in the classical theory of electro-magnetic field.

In the GD there is also gravitational field with the energy density θ_{00} (1) around any mass distributed in the spherically-symmetric way. As soon as the gravitational field energy density in vacuum (out of the source, i.e., out of a sphere filled by matter) becomes of the order of the mean rest energy of the system 'matter + field'

$$\theta_{00} = \frac{GM^2}{8\pi r^4} \sim \frac{Mc^2}{r^3} \qquad \text{for} \quad r \sim \frac{GM}{c^2} \quad , \tag{3}$$

in the GD the same question as in ED arises. Where, in that case, is the mass of such a 'point' object concentrated? Ultimately, what is the rest mass M of the gravitating body on the whole?

It is obvious that these questions arise at distances of the order of GM/c^2 - i.e., of the order of the gravitational radius of the 'point' source (or gravitating centre) as one can see from comparison (3). The estimation of the gravitational radius is made here by the same reasons as the estimation of the classical electron radius $e^2/m_e c^2$ in ED. Just as in the classical ED we may say that we deal with a theory much alike the classical linear ED until some point sources distribution is compressed to dimensions when distances between them become of the order of Gm_a/c^2.

For the spherically-symmetric distribution of points with the centre at the origin of coordinates it means that the all system (with the rest energy Mc^2) of such an 'elementary' gravitating object is *not* compressed to dimensions of the order of GM/c^2 or as long as the field is measured at distances much greater than GM/c^2. Thus, conventionally a 'point' in the GD is in fact something with finite dimensions of the order of GM/c^2. It means that at $r >> GM/c^2$ the mass M of the point source includes automatically the 'mass' of the gravitational field itself generated by the source. This is the sense of 'point' idealization in the GD.

It is now obvious that nonlinear GD is the GD at distances of the order of gravitational radius ($\sim GM/c^2$) from the centre of any spherically-symmetric configuration. In accordance with the universal character of gravitational interaction one must consider the field itself to be the source of gravitational field (there is nothing of the kind in ED). On the right side of the field equations it is accounted for by including the gravitational field EMT in sources (a kind of the source 'splitting' occurs) which makes the field equations nonlinear. Accordingly, corrections to potentials arise which lead in particular to an entire explication of the observed effect - the Mercury perihelion shift (a 'nonlinear' contribution in the effect).

Nevertheless, the question about the mass M in the nonlinear GD remains: where is the mass located if the gravitational field energy is positive, localizable and condition (3) is satisfied? The outer field of such a collapsar remains static as before. More precisely, the question concerns the outer static field of a compact spherically-symmetric configuration with dimension of the region filled by matter of the order of the gravitational radius of the whole system (GM/c^2). It is this object which we shall call the *collapsar* in what follows.

The following sections of this paper will show in more detail that it turns out that, to answer the questions formulated in the Introduction about static field of the collapsar, there is no necessity to resolve the field equations to all approximations. The essence of the matter can be cleared quite precisely in the following way. Let us assume that there is such a stationary state: the collapsar with the radius of the sphere filled by particles (matter) close to the gravitational radius of the whole system (GM/c^2). Let us integrate the gravitational field energy density θ_{00} in vacuum the integral going from the surface of the sphere filled by particles (with a radius r_x) to infinity and being equated to the total rest energy of the whole configuration

$$4\pi\int_{r_x}^{\infty}\frac{GM^2}{8\pi r^4}r^2dr = Mc^2 \quad . \tag{4}$$

This equality must be correct at some finite $r_x \sim GM/c^2$. But then it should be supposed that the total rest mass of the gravitating object is the 'mass' of the gravitational field only. The latter is difficult to reconcile with assumption (2) about the 'masslessness' of gravitational field - i.e., with the assumption on gravitons as on particles with the zero rest-mass. Strictly speaking, in that case it is difficult to maintain the assumption on static character of field around the collapsar.

Thus, proceeding from general principles (axioms) lying at the base of the relativistic field theory, one succeeds in formulating a statement (theorem) of the collapsar static field, the sense of which can be expressed in the following way:

If the gravitational field energy is positive ($\theta_{00} \geq 0$) and if the field is really 'massless' ($\theta^{ik}\eta_{ik} \equiv 0$) then the localization of gravitational field energy means that it is impossible to compress matter (particles) to a sphere with a radius smaller than some finite radius, still static spherically-symmetric field being in vacuum around this sphere.

Our paper is dedicated entire to development of a detailed and consistent basis of this statement. But first I return to basic equations of GD, commenting upon them in detail from the field-theoretical point of view. Then (and it is very important) the static field will be represented explicitly as a sum of two field components with equations for each of them: namely, a scalar component and a purely tensor component of the only gravitation field. The choice of equations of motion for test particles in a given gravitational field will be based on general principles. The so-called 'linear' contribution to the Mercury's perihelion shift will be regarded. Then the choice of formula (1) for θ_{00} will be based in detail and the gravitational field energy will be represented as suitable positive contributions of each of gravitation components, both scalar and purely tensor ones.

Ultimately, on the basis of the fact that the scalar component of gravitation by virtue of (2) is described by the linear equation down to $r \approx GM/c^2$, it will be shown that the radius of the sphere filled by matter with outer static field anyway cannot be less than ¼ GM/c^2, the contribution of each field component in θ_{00} being positive. In that way the question on the singularity is resolved in the GD.

3. The Linear Gravidynamics and The Scalar Component of Gravitation

A considerable part of what follows is based on ideas expressed already in papers by Thirring, Moshinsky, Hoopte, Fock, and others. It is evident that the relativistic theory of classical tensor field pretending to a complete description of gravitation will inevitably use the experience of GR by Einstein and of ED by Maxwell. For all this as we will see from the following that, developing the field-theoretic interpretation of gravitation, we adhere to the field (dynamic) interpretation of basic principles of this theory. In particular, we proceed from the fact that gauge transformations of potentials are not connected in any way with the known transformations of frame of references in GR. And the principle of invariance under the gauge transformations, rather than the principle of equivalence, must lie at the basis of the consistent field-theoretic approach to gravitation.

When we say about some field theory that, means in the first instance it represents the field equations (for example, Maxwellian equations) - just as the Einsteinian GR represents first of all the equations at the basis of this theory. But it is impossible to understand the usual form of the Einsteinian equations in the non-geometrical way. In latter there already is a curvature, i.e., the curved space-time. Ultimately it defines the geometrical description of test particle motions in gravitational field (i.e., in the curved space-time of GR), it defines famous geodesic equations. Everybody agrees that the dynamic interpretation of Einsteinian equations begins in case of rather weak fields.

By the linear GD I mean all cases when gravitational field can be assumed weak. In that case distances between particles ('gravitational charges') must be much greater than their gravitational radius, and gravitating points may be assumed to be real point structureless objects for which, in particular, the mass conservation law is fulfilled: i.e.,

$$\frac{\partial}{\partial x^k}(\mu \frac{dx^k}{dt}) = 0 \quad . \tag{5}$$

Where $\mu = \Sigma_\alpha m_a \delta(\mathbf{r} - \mathbf{r}_\mathrm{a})$ is the gravitating points mass density. Particles in linear the GD can have any velocities up to relativistic ones. The natural wish for generalization of the Newtonian gravitation up to relativistic velocities leads to this first (rather simple) part of GD, i.e., to the relativistic theory of gravitation in flat space-time (the relativistic gravidynamics).

Let us begin with the simplest case of a sourceless tensor field. As is known, a symmetric tensor of the second rank consistent with the Klein-Gordon theory - namely,

$$\Box \Phi_{ik} = 0 \tag{6}$$

describes a massless field (particles with spin 2) which we need, if the following conditions are satisfied:

$$\Phi^{ik}{}_{,k} = 0 \ , \tag{7}$$

$$\eta_{ik}\Phi^{ik} = 0 \ , \tag{8}$$

Equations (6) together with the 5 invariant conditions (7), (8) define the Φ_{ik} accurate up to the gauge transformation of the potentials

$$\Phi_{ik} \rightarrow \Phi'_{ik} = \Phi_{ik} + A_{i,k} + A_{k,i} \quad . \tag{9}$$

Where A_i is a vector field consistent with the conditions $\Box A_i = 0$ and $A^i{}_{,i} = 0$ (*here* $\Box\,\Phi_{ik} \equiv - \Phi_{ik,l}{}^{l}$).

One can construct (or postulate, if it is wanted) a more general form of a linear differential equation of second order for the symmetric tensor $\Psi_{ik} = \Psi_{ki}$, demanding directly an invariance of these equations under the transformation (9). We obtain eventually equations coinciding in form with those usually called in GR the 'linear form' of the Einsteinian equations (the left-hand side):

$$-\Psi^{ik,l}{}_{l} + \Psi^{il,k}{}_{l} + \Psi^{kl,i}{}_{l} - \Psi^{,ik} + \eta^{ik}(\Psi^{,l}{}_{l} - \Psi^{mn}{}_{,mn}) = 0 \tag{10}$$

For all this the numerical value of the factor (+1) before the brackets is fixed by the condition of identical equality of the left-side divergence to zero, which is a consequence of the same invariance under the gauge transformations group (9).

The consistent dynamic interpretation of these equations is that field potential Ψ_{ik} (just as in ED) is understood irrespective of the metric η^{ik}. In particular, there is no such a condition like $\Psi_{ik} \ll \eta^{ik}$. The potentials Ψ_{ik} by themselves have as little sense as 4-potentials in ED. They may have any value because of their uncertainty according to (9).

Generally speaking, one can write down the gauge (gradient) transformation group under which equation (10) must be invariant, in more general form (but which does not coincide outwardly with the well known group of infinitesimal transformation in GR):

$$\Psi_{ik} \rightarrow \Psi'_{ik} = \Psi_{ik} + A_{i,k} + A_{k,i} + \Lambda_{,ik} \quad , \tag{10a}$$

with 5 arbitrary functions A_i (4-vector) and Λ (scalar). Then to obtain equations like (6) from the general equations (10) (for particles/gravitons with spin 2 only) it is necessary to require the satisfaction of 5 gauge conditions like (7), (8) at once, for the 'gauging/calibrating ' scalar Λ and vector A^i fields satisfying the equations

$$\Box A^i + A^{l,i}{}_{l} = j^i \ ; \qquad \Box\Lambda = \Psi + 2A^l{}_{,l} \quad . \tag{11$'$}$$

Where $j^i \equiv \Psi^{il}{}_{,l} - \Psi^{,i}$ ($\Psi \equiv \eta_{ik}\Psi^{ik}$) and $j^i{}_{,i} = 0$ according to field equations (10).

The field Lagrangian, which the equations (10) are obtained from, is of the form:

$$Z = a(F^{lmn}F_{nml} - F^{mn}{}_{m}F_{ln}{}^{l}) + b(\Psi^{lm,n}\Psi_{nm,l} - \Psi^{mn}{}_{,m}\Psi_{ln}{}^{,l}) \quad , \tag{12}$$

where $F_{lmn} = \Psi_{ln,m} - \Psi_{lm,n}$ and the value of a depends on the choice of the measurement units for the potentials Ψ_{ik} (see below). The expression in the second bracket is reduced to the divergence and does not influence the field equations for any b. (By variation of the values of a and b one can obtain all Lagrangians met in literature which the same equations (10) follow from.)

The interaction between the field Ψ_{ik} and its source T^{ik} is described by the Lagrangian

$$\frac{f}{c^2}\Psi_{ik}T^{ik} \quad , \tag{13}$$

where f plays the role of an interaction constant or coupling constant and is defined specifically at the choice of the measurement units for Ψ_{ik} , and T_{ik} corresponds in the considered case of the linear GD to an EMT of the system of point (structureless) objects-particles. That is here

$$T^{ik} = \mu c u^i u^k \frac{ds}{dt} \quad , \quad ds = c\, dt\, \sqrt{1 - \mathrm{v}^2 / c^2} \quad ,$$

where u^i is the 4-velocity and v is the usual velocity of the particles. I emphasize once more that we consistently adhere to point idealization for gravitational field sources ($\mu = \Sigma_\alpha\, m_a\, \delta(\mathbf{r} - \mathbf{r}_{\mathrm{a}})$) as an initial notion of the theory.

One can obtain a general form of field equations with sources in the linear GD by virtue of the action

$$\frac{1}{c}\int (Z + \frac{f}{c^2}\Psi_{ik}T^{ik})d\Omega$$

(the sources motion is assumed to be specified), which leads to the field equations in the form of:

$$-\Psi^{ik,l}{}_{l} + \Psi^{il,k}{}_{l} + \Psi^{kl,i}{}_{l} - \Psi^{,ik} + \eta^{ik}(\Psi^{,l}{}_{l} - \Psi^{mn}{}_{,mn}) = -\frac{f}{2ac^2}\mathrm{T}^{ik} \quad . \tag{14}$$

Because of general character of these equations they cannot, however, to be applicable directly to solve some concrete problem; for example, to find the field of only one point source. Both the symmetric tensor Ψ_{ik} by itself and equations (14) describe a whole 'mixture' of fields - this is simultaneously scalar, vector and tensor fields. In that case they say about 'the mixture' of fields with different spins ($[\Psi_{ik}] = 0 \oplus 0 \oplus 1 \oplus 2$), see, for example, Sexl (1967). This property must also be attributed to the tensor source T^{ik} on the right-hand side of (14). But the identical equality of divergence of the left-hand side of these equations to zero, or, in other words, gauge invariance (10a), demands the fulfilment of the conservation law for the tensor source (so-called 'strong' law)

$$T^{ik}{}_{,k} = 0, \qquad (15)$$

as in ED the fulfilment of the conservation law for the 4-current $j^i{}_{,i} = 0$ is a consequence of the gauge (gradient) invariance of the Maxwell's equations. Equality (15) means that the vector component of the field Ψ_{ik} may be regarded as dependent since the corresponding vector source is absent (or, a vector field may be only virtual…).

Of course, in the linear GD approximation, when gravitational interaction between particles may be assumed weak, the equality (15) is fulfilled only approximately. An exact conservation law $(T^{ik} + \theta^{ik})_{,k} = 0$ will be true only for the sum of the source EMT $(T^{ik} + \theta^{ik})$. That is just a consequence of relativistic invariance of the field equation (14), but not the gauge one. But as was noticed in the previous section, in the linear GD the strong field as a source is automatically included in the notion of a 'point' source with some mass M (there is no 'splitting'). Then the field Ψ_{ik} in (14) is the field at distances much greater than GM/c^2 for each source in the right side (for the weak field), and equality (15) must be regarded as a consequence of gauge invariance of equations (14).

Thus, if we approximately regard the sources in the right-hand side of field equations (14) as the point sources with given masses, then, allowing for the gauge invariance of these equations, one can bind the vector component of the field Ψ_{ik} by the gauge condition (the Hilbert-Lorentz gauge condition):

$$\Psi^{im}{}_{,m} = ½\, \Psi^{,i} \quad , \qquad (16)$$

and below we regard the vector field in Ψ_{ik} to be dependent (so we 'exclude' the vector component of Ψ_{ik}). In that case, the field equations (14) are transformed to the form

$$\Box\, \Psi_{ik} = -\frac{f}{2ac^2}(T_{ik} - ½\, \eta_{ik}T) \quad , \qquad (17)$$

where $T = \eta_{mn}T^{mn}$ is a scalar presenting the trace of the sources EMT. It is impossible, in particular, to demand the satisfaction for Ψ_{ik} of the 5 invariant conditions (7), (8) at once ('to exclude' also the scalar Ψ) for equations (14) because of the fact that $T \neq 0$. [Just the nonzero trace of the sources EMT (the scalar) permits an assuming that the scalar Ψ cannot be always only a virtual field.]

The system of equations (17) still describes 'the mixture' of fields but the amount of components of this mixture is now less because of condition (16). And now we separate explicitly these components presenting equations (17) in the form of an equivalent equation system for each component separately - scalar and purely tensor ones.

Let us present the potential Ψ_{ik} in such an invariant form distinguishing explicitly between scalar and tensor components

$$\Psi_{ik} \equiv \Phi_{ik} + \tfrac{1}{4}\,\eta_{ik}\Psi \quad , \tag{18}$$

where $\Phi_{ik}\eta^{ik} \equiv 0$ and Φ_{ik} describes now the tensor component in the field Ψ_{ik}, as well as invariant tensor $\tfrac{1}{4}\,\eta_{ik}\Psi$ describes only scalar component.

In exactly the same way one can manipulate with T^{ik}:

$$T_{ik} \equiv T_{ik}^{(2)} + \tfrac{1}{4}\,T \quad , \tag{19}$$

where $T_{ik}^{(2)}\,\eta_{ik} \equiv 0$ for the sources $T_{ik}^{(2)}$ of the tensor component Φ_{ik} in the field Ψ_{ik}.

Convolution of equations (17) by indices yields an equation only for the scalar part of the field Ψ_{ik}. If one now substitutes expressions (18) and (19) in (17) and uses the equation for the scalar part then one can obtain an equation only for the tensor component of gravitation also. As a result we obtain

$$\Box\Psi = +\frac{f}{2ac^2}T \quad , \tag{20)*}$$

$$\Box\Phi_{ik} = -\frac{f}{2ac^2}T_{ik}^{(2)} \quad . \tag{21)*}$$

The Hilbert-Lorentz gauge condition (16), written down now in the form $\Phi^{mn}{}_{,m} = \tfrac{1}{4}\,\Psi^{,n}$, excludes as before the vector component of gravitation.

Thus we have formulated the basic equations in the linear GD which will be naturally generalized below for strong fields, i.e., when sources cannot be already represented as systems of only 'point' structureless objects and 'a splitting' of them into a 'field' part and a 'point' one will be needed. But the main conclusion from equations (20)* and (21)* is the following: gravitation in the GD has two components: scalar and tensor ones each interacting with its source with the same coupling constant *f*. In our opinion it is this condition which is the essence of the dynamic interpretation of equations of type (10) or (14), and this is a radical difference of the approach developed by us from different versions of alternative theories (the scalar-tensor one, bimetric formalism, etc.).

4. The Outer Field of a Massive Gravitating Centre at Distances of $r \gg GM/c^2$. Motion in Given Field

Section 2 has mentioned that in the consistent GD description it is convenient, at least at the beginning, to eliminate all interactions except the gravitational one. A gravitating 'body' in the right-hand side of the equations (*) (20* and 21*) is in fact a system of point sources interacting only gravitationally. All the more, in the linear GD one can obtain the field of any system of point sources (both inside the system and outside it) as a superposition of fields of such 'elementary' sources. Furthermore, the gravitational potentials of the point objects coincide with the potential of real spherically-symmetric bodies.

But we begin with gaining an understanding of what this spherically-symmetric and static field of only one 'elementary' motionless point source in the origin of coordinates is. One can obtain the EMT of the mass M point particle located in the origin of coordinates from the general expression for $T^{ik} = \mu c^2 u^i u^k \sqrt{1-\mathrm{v}^2/c^2}$, in which one must take for the particle with the mass M in the origin of coordinates

$$\mu c^2 = Mc^2\delta(\mathbf{r} - \mathbf{r}_M) = Mc^2\delta(\mathbf{r}).$$

Hence, for the point source in the centre we have

$$T^{ik} = Mc^2\ \delta(\mathbf{r})\ \mathrm{diag}(1, 0,0,0). \qquad (22)$$

Here the frame of reference is fixed at last: i.e., this is *the frame of reference* (inertial, of course) of a body of reference + generally speaking any frame of coordinates in which the source-particle (the body of reference) rests in the origin of coordinates ($\mathbf{r}_M = 0$ and $\mathrm{v} = 0$).

Since the source is at rest, the field must be static, centrally symmetric, and the system of the equations (*) will be rewritten as

$$\frac{1}{r}\frac{d^2}{dr^2}[r\Psi(r)] = +\frac{f}{2ac^2}T \quad , \qquad (23)$$

$$\frac{1}{r}\frac{d^2}{dr^2}[r\Phi_{ik}(r)] = -\frac{f}{2ac^2}T^{(2)}_{ik} \quad , \qquad (24)$$

For the scalar source T from (22) we have

$$T = \eta_{ik}T^{ik} = Mc^2\ \delta(\mathbf{r}) \quad , \qquad (25)$$

and equation (23) for the scalar field $\Psi(r)$ has the solution

$$\Psi(r) = -\frac{fM}{8\pi a}\frac{1}{r} \quad , \qquad (26)$$

consistent with the additional boundary condition: the scalar potential must be zero at infinity.

For the purely tensor source $T^{(2)}{}_{ik}$ from (19) and (22) we have (in case when $r >> GM/c^2$ or when the total mass of the source is assumed to be in the centre):

$$T^{(2)}{}_{ik} = \tfrac{3}{4}\, Mc^2\ \delta(\mathbf{r})\ \mathrm{diag}\ (1,\ 1/3,\ 1/3,\ 1/3) \quad . \qquad (27)$$

In that case the solution of equation (24) with the same boundary condition (the potential at infinity must be zero) will be the tensor field

$$\Phi_{ik}(r) = \Phi_{00}(r)\ \mathrm{diag}(1,\ 1/3,\ 1/3,\ 1/3)\ , \tag{28}$$

$$\Phi_{00}(r) = +\frac{3}{2}\frac{fM}{16\pi a}\frac{1}{r}\ , \qquad r >> GM/c^2\ .$$

Thus the static field of a massive point in the origin of coordinates is described by two potentials (which are functions of *r),* namely,

$$\Psi(r) = -\frac{C}{r} \quad \text{and} \quad \Phi_{00}(r) = +\frac{3}{2}\frac{C}{r}, \qquad \text{if} \quad C \equiv \frac{fM}{16\pi a}.$$

And one can unite it, if desired, in one single tensor field:

$$\Psi_{ik} = +\frac{3}{2}\frac{C}{r} diag(1,\frac{1}{3},\frac{1}{3},\frac{1}{3}) - \frac{1}{2}\frac{C}{r} diag(1,-1,-1,-1) =$$

$$= +\frac{fM}{16\pi a}\frac{1}{r} diag(1,1,1,1)\,. \tag{29}$$

And there is already no ambiguity or arbitrariness connected with the gauge invariance. The static potential (29) is determined quite uniquely (of course without allowing for radiation incident from outside) by gauge condition (16) and by the additional condition: Ψ_{ik} must be zero at infinity.

Evidently, the solution (29), so as (26) and (28), can be quite applicable as description of outer field outside the sphere of the radius r_0 filled by matter with any centrally symmetric distribution of mass $\rho(r)$. In that case one can take right away the EMT of a macroscopic body (as a continuous medium) in the form $\rho c^2 u^i u^k$ as the source in the right side of field equations. Here ρ is now the body mass density and u^i is some element average 4-velocity of (*macroscopic*) 'elementary' volumes of averaging. For the static case or if macroscopic motions are slow, the EMT of a gravitating body will be of the form $\approx \rho c^2$ diag(l, 0, 0, 0). Solving the field equations one can find both an inside solution and an outside one. The outside solution for any $r \geq r_0$ will be determined only by the total mass inside the sphere $r = r_0$ and will coincide with the solution (29) simply because of the Poisson's equation property for centrally symmetric $\rho(r)$:

$$4\pi\int_0^{r_0} \rho(r) r^2 dr = M = \int_{V_0} M\delta(r) dV\ ,$$

where V_0 is a volume of the sphere with the centre in the origin of coordinates. In that case (as in the Newtonian gravitation) the dimension of the gravitating body is of no importance, of course, till the configuration is negligible in comparison with the total rest energy Mc^2.

In accordance with the more consistent discrete conception of gravitating bodies accepted here (in the GD), it means that 'points' with masses m_a inside the body are located at distances from each other much greater than Gm_a/c^2. Hence, till this configuration is quite far from the dimension of the order of GM/c^2 and the masses m_a of the 'points' are the masses of rather large (macroscopic) regions indeed, then an antipressure needed to secure the stability of such a sphere is simply an elasticity of matter which these regions consist of. In other words, here some nongravitational interaction is meant which defines the equation of matter state.

Let us now consider the motion in a given field. As was noticed before, the interaction of the field Ψ_{ik} with particles is described by the scalar $(f/c^2)\Psi_{ik}T^k$ (13) and one can obtain the particles motion in field from the action

$$S = \frac{1}{c}\int(-\eta_{ik}T^{ik} + \frac{f}{c^2}\Psi_{ik}T^{ik})d\Omega \quad , \tag{30}$$

which gives for test point particle with the mass m

$$S = \int(-mcds + \frac{fm}{c}\Psi_{ik}u^i dx^k) \quad . \tag{31}$$

If we compare this with the action for a charged particle in a given electromagnetic field and introducing the 4-vector

$$A^{(g)}{}_k \equiv -\Psi_{ik}u^i \quad , \tag{32}$$

we can write the action for a particle in a given gravitational field in the 'electrodynamic' form

$$S = \int(-mcds - \frac{e}{c}A_k^{(g)}dx^k) \quad , \tag{31)'$$

where $e \equiv fm$. A variation of (31) or (31)' (as in Landau and Lifshitz, 1973; and see detail in paper by Baryshev and Sokolov, 1983) gives equations of motion that are convenient also to be written down by the vector $A_k^{(g)}$ in the same 'electrodynamic' form

$$mc\frac{du^i}{ds} = \frac{e}{c}(A_{k,i}^{(g)} - A_{i,k}^{(g)})u^k \quad . \tag{33}$$

To emphasize that in GD one can use explicitly the conception of force, as in ED, one can even introduce the 3-dimensional vectors

$$\mathbf{E}^{(g)} \equiv -\frac{1}{c}\frac{\partial \mathbf{A}^{(g)}}{\partial t} - \nabla\varphi^{(g)} \quad , \qquad \mathbf{H}^{(g)} \equiv \mathbf{A}^{(g)} \quad ,$$

where $\varphi^{(g)} \equiv -A_0$, $\mathbf{A}^{(g)} = -A_\alpha$ (α = 1, 2, 3). Then 3-dimensional equations of motion from (33) coincide by form with the Lorentz force and with the equation for energy in ED

$$\frac{d\mathbf{p}}{dt} = \mathrm{e}\,(\,\mathbf{E}^{(g)} + \frac{1}{c}[\mathbf{v}\cdot\mathbf{H}^{(g)}]\,)\,; \qquad \frac{dE_{kin}}{dt} = \mathrm{e}(\mathbf{E}^{(g)}\cdot\mathbf{v}) \;. \tag{34}$$

The fact that in (30) T_{ik} enters both the 'inertial' part of action and the 'gravitational' one, is a direct consequence (or rather a generalization) of equality of the inertial mass m_{inert} of particles and the gravitational one m_{grav} ($m_{inert} = m_{grav}$). Thus, if from the very beginning we proceed directly from the experimentally tested law of equality of the inertial mass and the gravitational one for test bodies understood in GD *as point* (structureless) objects but not from the 'equivalence principle', then equations (33), introduced for the first time in GD by Birkhoff (1944), can be a perfectly relativistic generalization of the Newton's equations of motion

$$\dot{\mathbf{v}} = -\nabla\varphi_N \frac{m_{grav}}{m_{inert}} \quad ,$$

where $m_{grav} = m_{inert} \equiv m$.

One can now write an expression for a force acting on a motionless test particle located in the field (29), explicitly presenting this force as the sum of forces: $\mathbf{F}_{(2)}$ is the force connected with the tensor component of field (28) and $\mathbf{F}_{(0)}$ is the force connected with the scalar component (26). To do that it is convenient to use the 4-vector $A^{(g)}{}_k$ (32) and the equation of motion from (34). If the particle is at rest or, generally speaking, moving but very slowly ($v/c \sim 0$), then for each term in (29) only 0-component of correspondent 4-vectors will be nonzero. I.e. (further the symbol g is omitted for $\varphi^{(g)}$)

$$\varphi_{(2)} = -\frac{3}{2}\frac{C}{r} \quad \text{and} \quad \varphi_{(0)} = \frac{1}{2}\frac{C}{r} \quad .$$

Correspondingly, for the forces we have

$$\mathbf{F}_{(2)} = -e\frac{d\varphi_{(2)}}{dr}\frac{\mathbf{r}}{r} \qquad \text{and} \qquad \mathbf{F}_{(0)} = -e\frac{d\varphi_{(0)}}{dr}\frac{\mathbf{r}}{r} \quad .$$

And if we connect the constants f and a with the Newtonian gravity constant G like this: $G \equiv f^2/16\pi a$, one can write for each of forces separately

$$\mathbf{F}_{(2)} = -\frac{3}{2}\frac{GmM}{r^2}\frac{\mathbf{r}}{r} \quad \text{and} \quad \mathbf{F}_{(0)} = -\frac{1}{2}\frac{GmM}{r^2}\frac{\mathbf{r}}{r} \quad . \tag{35'}$$

It produces as a result the usual good old Newtonian law :-)

$$\mathbf{F} = \mathbf{F}_{(2)} + \mathbf{F}_{(0)} = -\frac{GmM}{r^2}\frac{\mathbf{r}}{r} \ , \tag{35}$$

i.e., the attraction in the sum.

Thus, here I would like to emphasize that even at $r >> GM/c^2$ the static, spherically-symmetric gravitational field in vacuum, generated by a massive object in the centre (in particular, by a quasi-static, centrally-symmetric distribution of mass with the density $\rho(r)$) as the physical action on test bodies is an algebraic sum of the attraction $\mathbf{F}_{(2)}$ (the gravitational *tensor* component proper) and the repulsion $\mathbf{F}_{(0)}$ (the *scalar* component of gravitation). The foregoing has shown that this property follows the most general principles lying in the base of the dynamic interpretation of the gravitational field. Or rather (35) must be considered as a direct consequence of the conclusion formulated at the end of the previous section for the case of weak field.

A description of interaction between a particle of a given rest mass m moving with some velocity v, and centrally-symmetric field (29) of a motionless massive centre, can be obtained from action (31) in which the quantity

$$L = -mc^2\sqrt{1-\mathrm{v}^2/c^2} + fm\Psi_{00}\frac{(1+\mathrm{v}^2/c^2)}{\sqrt{1-\mathrm{v}^2/c^2}} \tag{36}$$

is an analogue of the Lagrange's function with $\Psi_{00} \equiv C/r$. An explicit separation of the test particle interaction with both components of the field allows writing down L in the form

$$L = -mc^2\sqrt{1-\mathrm{v}^2/c^2} + fm\Phi_{00}\frac{(1+\mathrm{v}^2/3c^2)}{\sqrt{1-\mathrm{v}^2/c^2}} + \frac{1}{4}fm\Psi\sqrt{1-\mathrm{v}^2/c^2} \ . \tag{36)'$$

The equations of motion of the particle in the centrally-symmetric field accurate up to terms of the order of $\mathrm{v}^2/c^2 << 1$ follow directly the Lagrange's function

$$L = (\frac{m\mathrm{v}^2}{2} - m\varphi_N) + (\frac{1}{4}\frac{m\mathrm{v}^2}{2} - \frac{3}{2}m\varphi_N)\frac{\mathrm{v}^2}{c^2} \ , \tag{37}$$

$$L = (\frac{m\mathrm{v}^2}{2} + fm\Phi_{00} + \frac{1}{4}fm\Psi) + (\frac{1}{4}\frac{m\mathrm{v}^2}{2} + \frac{5}{6}fm\Phi_{00} - \frac{1}{8}fm\Psi)\frac{\mathrm{v}^2}{c^2} \ ,$$

the expression for which can be obtained from (36) by expansion into a power of (v/c) series up to terms of the second order inclusive. And it was taken into account here that (v^2/c^2) and (φ_N/c^2) are terms of an equal infinitesimal, when the particle moves by action of gravitational field only. (A small addition in (37) to the classical Lagrange's function only depends ultimately on r, if v is expressed by the velocity of the undisturbed Kepler problem.)

Here we come close to the explication of the Mercury perihelion shift effect in GD connected, in particular, with the choice of θ_{ik} - the EMT gravitational field. In GD this effect is in fact an algebraic sum of two effects:

(1) The first (*linear*) effect arises because relativistic corrections of the order of v^2/c^2 are allowed at the test particle motion in the static field of a massive centre. In point of fact this effect is connected a relativistic lag of gravitational interaction between the Sun and the Mercury at motion of the latter by its orbit.

(2) The second (*nonlinear* proper) effect is connected with corrections of the tensor potentials (Φ_{ik}) of a massive centre (corrections of the temporal component Φ_{00}) which arises when allowing gravitational field continuously distributing *positive* energy around the Sun. It is this part of the effect that depends on the correct choice of the formula for θ_{00}, which will be the question in the next section.

Here we only explain the first effect arising due to the test particle interaction with the field (29) (with the 2 components). Considering the Keplerian motion ($m << M$) with a small addition

$$\left(\frac{m\mathrm{v}^2}{8} - \frac{3}{2} m\varphi_N\right)\frac{\mathrm{v}^2}{c^2}$$

to the classical Lagrange's function we obtain the value of the perihelion shift $\delta\varphi_1$ equal to

$$\delta\varphi_1 = \frac{7\pi}{(1-e^2)}\frac{GM/c^2}{a} \quad . \tag{38}$$

In this formula (for the perihelion shift only) a is the usual semi-major axis designation of the orbit and e – orbit eccentricity.

Other effects (the light deflection in the solar field, the lag of a radio signal in the same field, the gravitational drift of atom frequencies = the gravitational red shift) are considered in detail in the paper by Mosinsky (1950) and in the paper by Baryshev and Sokolov (1983) using the same nomenclature that is accepted here. The light deflection and the signal lag (i.e., the interaction between electromagnetic field and a given gravitational field) is understood in GD as interaction between light and a non-homogeneous matter 'medium' with the refraction index

$$n = 1 + 2\frac{GM/c^2}{r}$$

and, correspondingly, with the velocity of light propagation in this 'medium' $c_g = c/n$. Both this effects arise because of the interaction *only* with the tensor component of gravitation since the corresponding scalar in

$$\frac{f}{c^2}\Psi_{ik}T^{ik}_{(el)}=\frac{f}{c^2}\Phi_{ik}T^{ik}_{(el)}+\frac{f}{c^2}\frac{1}{4}\Psi\eta_{ik}T^{ik}_{(el)}\equiv\frac{f}{c^2}\Phi_{ik}T^{ik}_{(el)}$$

is identically equal to zero ($\eta_{ik}T_{(el)}{}^{ik}\equiv 0$) to account of the property of $T_{(el)}{}^{ik}$ - the EMT of electromagnetic field.

A rigorous description of the gravitational frequency shift demands consideration of the atoms behaviour in gravitational field. The cause of the frequency change (the gravitational red shift) of a photon radiated by an atom is *the shift of the atom quantum levels* as a result of electromagnetic and spinor fields interactions with a given gravitational field (Mosinsky, 1950). The result of a rigorous analysis gives the effect value equal to $\Delta\nu/\nu = - GM/c^2r$ which coincides with the experimentally measured one.

5. Energy of the Central Source Static Gravitational Field

This section shows in detail why formula (1) for θ_{00} was chosen as the temporal component of the spherically-symmetric field EMT of any massive gravitating centre. The energy of each component of gravitation (Ψ and Φ_{ik}) will be found separately.

The EMT of gravitational field (without gravitational charges in it) was found in the GD in the paper by Sokolov and Baryshev (1980) where we used the following limitations as additional conditions for its choice: l) the obtained EMT must be symmetric ($\theta_{ik} = \theta_{ki}$); 2) θ_{ik} must have the trace ($\eta_{ik}\theta^{ik}\equiv 0$) identically equal to zero, whith is connected with the 'masslessness' of gravitational field; 3) it must always give the *positively* determined gravitational field energy density ($\theta_{00}\geq 0$); 4) the last condition concerns each of its components separately (i.e., $\theta_{(0)}{}^{00}\geq 0$ and $\theta_{(2)}{}^{00}\geq 0$).

But general principles alone are apparently not sufficient for the gravitational field energy density value final choice. To the opinion of investigators who tried to look at this question (Thirring, 1961; Sexl, 1967), the account of the positive gravitational field energy continuously distributed in space around a central source gives an appreciable contribution comparable by order of magnitude with observations of planets orbit perihelion shifts $\delta\varphi$ in the Sun gravitational field. It is the effect which cannot be described by allowing for only the relativistic lag of gravitational interaction. In other words, as was noted in quoted papers, the effect of the Mercury perihelion shift demands consideration of the so-called gravitational potential 'nonlinear' correction, arising because of account of the gravitational field EMT itself in the equations (20*, 21*) right-hand side. Naturally, we used this circumstance and chose in a sense the simplest expression for the EMT of the possible ones satisfying 4 conditions mentioned above, and which leads (as it turned out) to explication of the observed Mercury perihelion shift for 100 years. I.e., we directly used the fact that in GD the choice of θ_{ik} is limited by experiment too.

To obtain the EMT (in particular, in the so-called 'canonical' form) one must proceed from the gravitational field Lagrangian (12) alone. And one must take into account that the vector component of the field Ψ_{ik} is absent in it, i.e., the Hilbert-Lorentz condition $\Psi^{im}{}_{,m} = ½\ \Psi^{,i}$ (16) is satisfied. Besides, to obtain the canonical EMT in a *symmetric form* at once, one must choose the constant b in the field Lagrangian (12) in front of a negligible divergence addition like this: $b \equiv a$. Thus, the Lagrangian (12) is reduced to the form

$$Z = a\Psi_{mn,l}\Psi^{mn,l} - a\ ½\ \Psi_{,m}\Psi^{,m}\ ; \qquad (12)'$$

or, if to separate explicitly the purely tensor Φ_{ik} and scalar Ψ components of the field Ψ_{ik} (if $\Phi_{ik}{}^{,k} = ¼\ \Psi_{,i}$) we obtain

$$Z = a\Phi_{mn,l}\Phi^{mn,l} - a\ ¼\ \Psi_{,m}\Psi^{,m}\ , \qquad (39)$$

Hence, we have for the symmetrical (and canonical) EMT

$$t^{ik} = 2a(t^{ik}{}_{\Phi} - ¼\ t^{ik}{}_{\Psi})\ , \qquad (40)$$

where

$$t^{ik}{}_{\Phi} = \Phi^{mn,i}\Phi_{mn}{}^{,k} - ½\ \eta^{ik}\Phi_{mn,l}\Phi^{mn,l}\ ,$$
$$t^{ik}{}_{\Psi} = \Psi^{,i}\Psi^{,k} - ½\ \eta^{ik}\Psi^{,m}\Psi_{,m}\ .$$

If now one calculates the value of t^{00} for the gravitational field around a massive centre with the use of (26) and (28), we obtain already the result coinciding with the formula (1): $t^{00} = (\nabla \varphi_N)^2/8\pi G$.

However, the point is that the definition of the canonical EMT as a conserved quantity is still ambiguous: why exactly this EMT and not any other one? In particular, by means of a certain arbitrariness at the choice of the EMT one can redefine so (see, for example, Medvedev, 1977, p. 206) that a new tensor should have the trace identically equal to zero, which corresponds more to properties of the field under discussion. Thus, a hope appears to choose the unique EMT from an infinite set of various ones with the help of another general condition.

And however, if to redefine now at once the EMT into form (40) with Lagrangian (39) so that a new tensor should have no trace, then as a result we shall obtain the field energy density with (generally speaking) an uncertain sign. In particular, for the static field we come to an absurd result: energy of this field is equal to zero everywhere. (The fields Φ_{ik} and Ψ are still connected by the condition $\Phi_{ik}{}^{,k} = ¼\ \Psi_{,i}$, see below…)

If we calculate the field EMT, it is more logical to proceed from the notion of *free* field, e.g. away from its source. But for an interacting field (with its source) one may not speak by definition about any conserved quantities, including the conserved complex of the field energy-momentum. Only for the pure case of free field one can introduce dynamic (or conserved) quantities characterizing the system. These are like quantum numbers. Hence, it follows from the notion of the free field of a definite spin meaning that we deal with the field Ψ_{ik} which is a mixture of the purely tensor and scalar free fields (but interacting with matter by the same coupling constant in general).

Besides, the energy density of static gravitational field around a massive object which is ultimately to be calculated by means of the field EMT obtained expression, is most likely to be unchangeable and equal to $(\nabla\varphi_N)^2/8\pi G$. In spite of ambiguity of the field EMT canonical definition, its temporal component (in accordance with its physical sense) must uniquely determine some certain part of the system 'matter + field' total energy *(Mc^2)*, but outside the sphere filled by matter. (Here, in the GD, we can proceed from the notion of field energy accepted in ED...) As it will be shown below, the corresponding addition to the Mercury perihelion shift is just connected with the fact that a certain part of system energy (energy outside the sphere of the radius r around the Sun) is as if 'excluded', and at the distance r from the Sun (in vacuum) an effective decrease of the massive central body mass occurs by the factor of $(1 - ½\, GM/rc^2)$.

So, the equations of free gravitational field away from its sources are obtained from equations (20*, 21*):

$$\left.\begin{array}{r} \Box\Psi = 0 \\ \Box\Phi_{ik} = 0 \end{array}\right\} \qquad (41^{**})$$

Far from the sources one may completely assume the field Ψ and Φ_{ik} to be independent and free, then one can obtain each of the equations noted by (41**) by independent variation of corresponding parts in Lagrangian (39).

But if the fields Ψ and Φ_{ik} are really independent then, generally speaking, they must not already be scalar and tensor parts of some unique tensor $\Psi_{ik} = \Phi_{ik} + ¼\, \eta_{ik}\Psi$ ($\Phi_m^{\ m} \equiv 0$) consistent with the Hilbert-Lorentz condition $\Psi_{ik}^{\ ;k} = ½\, \Psi_{,i}$, which connects two fields Ψ and Φ_{ik} ($\Phi_{ik}^{\ ,k} = ¼\, \Psi_{,i}$). Such a connection is natural in (12)' or in (39), when the fields Ψ and $\Phi_{ik:}$ interact with its sources.

The third section has mentioned already that when considering the free field in vacuum one can, even on the level of Lagrangian (12) with field equations (10) by means of gauge transformation (10a), separate himself from scalars and vectors, independently of anything, demanding the satisfaction of 5 gauge conditions (7) and (8) (with the help of the arbitrary scalar Λ and vector A^i) to obtain equation $\Box\,\Phi_{ik} = 0$ *at once.* That is to say, in vacuum for the free field (i.e. without direct interaction with sources of field) we have the right to demand independently the satisfaction of the

5 conditions for the tensor field Φ_{ik} only right away in form

$$\Phi^{ik}{}_{,k} = 0 \quad \text{and} \quad \eta_{ik}\Phi^{ik} = 0. \tag{41}$$

The first (vector) condition is true now *not* because of the equation $\square\ \Psi = 0$ for the scalar field (as if the connection $\Phi^{ik}{}_{,k} = ¼\ \Psi^{,i}$ exist as before) but directly as the gauge condition *excluding the vector field*. In conformity with this pure case, one can obtain both the field equations and the EMT of the tensor component from Lagrangian

$$a\Phi_{mn,l}\Phi^{mn,l} \ , \tag{42}$$

which follows from (12) and fulfilment of conditions (41). Hence, the EMT of such a free field (of the determined spin) can be obtained from the corresponding part of (40) if to redefine the canonical EMT (see details in Medvedev, 1977; Sokolov and Baryshev, 1980) so that it should have the trace identically equal to zero (and owing to $\square\ \Phi_{ik} = 0$ too):

$$\theta^{ik}{}_{(2)} \equiv 2at^{ik}{}_{(2)} \ , \tag{43}$$

where

$$t^{ik}{}_{(2)} = 2/3\ \Phi_{mn}{}^{,i}\Phi^{mn,k} - 1/6\ \eta^{ik}\Phi_{mn,l}\Phi^{mn,l} - 1/3\ \Phi_{mn}\Phi^{mn,ik}$$

The independent scalar field Ψ is by no means excluded because of gauge conditions (41) for the purely tensor field. As follows from the equation (20*) in the system (20*, 21*), in principle the scalar field can be radiated by the corresponding part (the coupling constant f is the same) of the same source and afterwards become the free field independently of Φ_{ik} (in the linear GD). That cannot be said of the vector field which is not radiated at all because of conservation law (15), i.e., because of the vector source absence.

The possibility of radiation of scalar (longitudinal) waves is a separate question, it will be mentioned below. It is now important that the equation for a free scalar field $\square\Psi = 0$ in (41**) is obtained from the second term in (39), both the value and the constant sign to be not essential. It is necessary only that the scalar and tensor in (39) should be measured ultimately in identical units. This is connected once more with the genetic connection essence of these fields as both components of gravitation field interacting with matter by the same coupling constant f.

But for obtaining the EMT of the independent and free scalar field the negative constant before its Lagrangian in (39) does not fit. First of all, it leads to negative energy of the Ψ-field and, second (as it was said above), in sum with energy of the tensor field Φ_{ik} it gives an uncertain sign of gravitation field energy in whole.

Hence, for the positive definiteness of energy one must take the free scalar field Lagrangian in the form

$$+ a\, ¾\, \Psi_{,m}\, \Psi^{,m} \,, \tag{44}$$

with the same equation for free scalar field $\Box\Psi = 0$ (without a direct interaction with sources). The selection of the constant ¾ is connected *here* with the requirements (the axiom also) mentioned above, i.e., that the field energy density in vacuum or around a massive gravitating centre must coincide with one which the canonical (also symmetrical) EMT (40) gives in that case. Hence, one can obtain for the EMT of the scalar field with the trace identically equal to zero

$$\theta^{ik}{}_{(0)} \equiv 2\, at^{ik}{}_{(0)} \;, \tag{45}$$

where

$$t^{ik}{}_{(0)} = 1/2\, \Psi^{,i}\Psi^{,k} - 1/8\, \eta^{ik}\Psi^{,m}\Psi_{,m} - 1/4\, \Psi\Psi^{,ik} \;.$$

Thus, the EMT of gravitational field is a sum of tensors concerning the scalar and tensor components of the field

$$\theta^{ik} = \theta^{ik}{}_{(2)} + \theta^{ik}{}_{(0)} \equiv 2a(t^{ik}{}_{(2)} + t_{ik(0)}) \;. \tag{46}$$

This sum is consistent with the above mentioned requirements (axioms): $\theta^{ik} = \theta^{ki}$, $\theta^{m}{}_{m} \equiv 0$, $\theta_{00} \geq 0$, $\theta_{(0)}{}^{00} \geq 0$ and $\theta_{(2)}{}^{00} \geq 0$ (one can become sure of the latter by direct calculation). But besides, everywhere we adhere to the idea that it is necessary to proceed not only from Lagrangians whose choice is always ambiguous by definition (as is seen in (12)), but from the field equations which follow them and which are tested afterwards in experiments.

From (43) and (45) one can now obtain for the static field tensor and scalar components (28), (26) for a central source:

$$t^{00}_{(2)} = -\frac{1}{6}\Phi_{mn,l}\Phi^{mn,l} = \frac{1}{2}\frac{C^2}{r^4}, \quad t^{00}_{(0)} = -\frac{1}{8}\Psi_{,m}\Psi^{,m} = \frac{1}{2}\frac{C^2}{r^4}, \quad \text{with} \quad C \equiv \frac{fM}{16\pi a}.$$

One more *property* must, therefore, be added for the found field EMT to the properties mentioned above. It turns out that (at least when $r >> GM/c^2$) field energies of each component separately are equal to each other in the same space point

$$\theta^{00}_{(0)} \equiv \theta^{00}_{(2)} = \frac{1}{2}\frac{(\nabla\varphi_N)^2}{8\pi G} = \frac{1}{16\pi}\frac{GM^2}{r^4} \quad , \tag{47}$$

if the connection $G \equiv f^{\,2}/16\pi a$ for the constants G, f and a follows from Newtonian law (35). Property (47) for fields (26) and (28) is connected to a certain extent with the choice of the gravitational field EMT. The fundamental character property $\theta^{00}{}_{(0)} \equiv \theta^{00}{}_{(2)}$ (47) is to be tested in future. But of principle is the fact that in that case the planet orbit perihelion shifts $\delta\varphi$ is completely accounted for.

Let us return again to basic equations (20*, 21*) with sources in connection with the main aims of this article: the static collapsar field in GD, the energy problem for gravitational field, and the nonlinear GD. (What is the rest mass of a gravitating object at all and where is it concentrated?) Evidently, when approaching the gravitating centre down to $r \approx GM/c^2$ one must take into account that the total energy (Mc^2) of the system 'matter + gravitational field' consists of field energy in vacuum and energy inside the sphere filled by matter (particles). Here a simple 'point' notion of the central gravitating object ceases to be applicable and one must already account for a kind of 'a splitting' of sources in equations (24).

First let us consider a static centrally symmetric field $\Psi(r)$. Scalar source (25) in equation (23) remains always the same (i.e., does not 'split') when approaching the centre from $r >> GM/c^2$ down to $r \approx GM/c^2$. At least the gravitation alone in vacuum around a source cannot change this source because identical equality to zero of the gravitational field EMT trace ($\theta^m{}_m \equiv 0$ for massless field). Hence, it follows that equation (23) is linear down to the limit $r \approx GM/c^2$. It means that both the potential $\Psi(r)$ and the force (of repulsion) $\mathbf{F}_{(0)}$ and the energy density $\theta^{00}{}_{(0)}(r)$ are found exactly for all r down to $r \approx GM/c^2$.

Let us now consider a static, centrally symmetric tensor field $\Phi_{ik}(r)$. When moving from regions with $r >> GM/c^2$ closer and closer toward a central object with $r \approx GM/c^2$, a simple point approximation for purely tensor source $T^{ik}{}_{(2)}$ (27) must be satisfied from bad to worse (the source is splitted, i.e., one may not already use the same M in (27), the mass of the central 'point' as if reduced when approaching $r \approx GM/c^2$). When approaching the centre, the traceless tensor θ_{ik} (46) or the energy-tension of gravitational field itself continuously distributed in vacuum around a central source (body) must play an ever increasing role in vacuum as an additional source. Hence, equations (24) for purely tensor field $\Phi_{ik}(r)$ become nonlinear, which is different from linear equation (23) for the scalar component $\Psi(r)$.

If we use the results of this section one can calculate the EMT for field (29) in some point $\mathbf{r}$ for some given direction of Cartesian axes X, Y, Z:

$$\theta_{ik} = 2a\frac{C^2}{r^4}\begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & & & \\ 0 & & (\delta_{\alpha\beta} - 2\dfrac{x_\alpha x_\beta}{r^2}) & \\ 0 & & & \end{vmatrix},$$

where $\alpha, \beta = 1, 2, 3$; $\delta_{\alpha\beta} = 0$ $(\alpha \neq \beta)$, $\delta_{\alpha\beta} = 1$ $(\alpha = \beta)$. But since the choice of Cartesian axes direction in the same frame of reference for every point **r** is essentially arbitrary because of central symmetry of the problem under consideration, then an averaging over all equiprobable (having equal probability) directions of axes *X, Y,* Z, drawn from the centre, gives $\overline{x_\alpha x_\beta} = 0$, if $\alpha \neq \beta$ and $\overline{x_\alpha^2} = r^2/3$.

Thus, the energy-tension of gravitational field of the centrally-symmetric problem in every point located at a distance r from the origin of coordinates is given independently of axes direction by the tensor

$$\theta_{ik} = \theta_{00}\,\mathrm{diag}(1,\ 1/3,\ 1/3,\ 1/3), \quad \theta^m{}_m \equiv 0 \ . \tag{48}$$

In such a form the gravitational field EMT around a gravitating centre corresponds to some medium as if consisting of a relativistic gas (of virtual gravitons).

Since here the question is all over on field in vacuum around a sphere of $r \approx GM/c^2$, then we exclude for the present this small region $\delta r \sim GM/c^2$ in the centre. Then the equations for the potential $\Phi_{ik}(r)$ in vacuum will be

$$\frac{1}{r}\frac{d^2}{dr^2}[r\Phi_{ik}(r)] = -\frac{f}{2ac^2}\theta_{ik} \ . \tag{49}$$

Consequently, for 00-component of this equation we have

$$\frac{1}{r}\frac{d^2}{dr^2}[r\Phi_{00}(r)] = 0 - \frac{f}{c^2}\frac{C^2}{r^4} \ , \tag{49)'$$

outside the sphere $\delta r \sim GM/c^2$. A sum of the integral of this equation without the right side plus a solution of equation (49) with the right side will be a centrally symmetric, time-independent solution of this equation. Then we have

$$\Phi_{00}(r) = \mathrm{a} + \frac{\mathrm{b}}{r} + \left(-\frac{CGM/c^2}{2r^2}\right) \ .$$

Evidently, at $r >> GM/c^2$ the obtained potential must pass into potential (28), which defines (together with the $\Psi(r)$) the density θ_{00} in (49)'. On the other hand, we require, as before, the satisfaction of the old condition about the potential to be zero at $r \to \infty$. As a result, for the constants a and b we have a ≡ 0, b ≡ $C3/2$, and for the corrected potential $\Phi_{ik}(r)$ we obtain ultimately

$$\Phi_{00}(r) = \frac{3}{2}\frac{fM}{16\pi a}\frac{1}{r}\left(1 - \frac{1}{3}\frac{GM/c^2}{r}\right); \tag{28)'$$

$$\Phi_{ik}(r) = \Phi_{00}(r)\,\mathrm{diag}(1,\ 1/3,\ 1/3,\ 1/3) \ .$$

Here we can return to the unique tensor field with the following non-zero components

$$\Psi_{00}(r) = \frac{C}{r}\left(1 - \frac{1}{2}\frac{GM/c^2}{r}\right) \equiv \frac{GM(1 - GM/c^2/2r)}{fr} \quad ;$$

$$\Psi_{11} = \Psi_{22} = \Psi_{33} = \frac{C}{r}(1 - \frac{1}{6}\frac{GM/c^2}{r}) \quad . \tag{29'}$$

At calculation of a corresponding contribution $\delta\varphi_2$ into the perihelion shift only the correction for Φ_{00}-component of the potential Ψ_{ik} gives ultimately an essential addition to the classical Lagrange's function. Other small corrections (to Ψ_{11}, Ψ_{22}, Ψ_{33}) give an addition to the Lagrange's function proportional to v^2/c^2, which leads to a negligible contribution into the effect. Thus, a nonlinear contribution to the Mercury perihelion shift turns out to be equal to

$$\delta\varphi_2 = \frac{-\pi}{(1-e^2)}\frac{GM/c^2}{a} \quad , \tag{50}$$

which in sum with the linear effect of gravitational interaction lag (38) gives the well-known GR result

$$\delta\varphi = \delta\varphi_1 + \delta\varphi_2 = \frac{6\pi}{(1-e^2)}\frac{GM/c^2}{a} \quad .$$

(In this formulas for the perihelion shift a is the semi-major axis of the orbit and e its eccentricity, as was in (38).)

The sign of the nonlinear perihelion shift $\delta\varphi_2$ is evidently connected directly with the sign of the gravitational field energy θ_{00}, as was seen from the foregoing. And the nonlinear contribution into the effect of Mercury perihelion shift is completely connected only with the corrections to the tensor component of gravitation.

6. Nonlinear Gravidynamics and the Theorem on the Collapsar Static Field

Within the bounds of the linear GD one can also predict new (respective to GR) effects in the weak field of the Earth. The description of such effects in the experiment with a gyroscope on orbit of the Earth, which could differ dynamic interpretation of gravitation (GD) from the geometrical one (GR), is accounted in the paper by Baryshev and Sokolov (1983).

But the most essential difference from GR mentioned above can be the existence in GD of the so-called scalar component of gravitation. In particular, the possibility of scalar radiation or longitudinal gravitational waves emerging, for example, at spherically-symmetric pulsations and spherically-symmetric collapse of a gravity field source, follows equation (20*) in the system (20*, 21*). The possibility in principle of such a radiation in GD allows approaching absolutely otherwise (than in GR it was) to description of the very process of a relativistic, spherically-symmetric gravitational collapse. Such a collapse should be understood in GD as a process (catastrophic may be) of the system 'particles + field' transition into a more and more bound state at which the system loses a part of its energy - rest mass - in the form of longitudinal (scalar) gravitational waves.

At the collapse of spherically-symmetrical distributed matter (or a particles system) and at formation of a maximum bound body with the dimension of order of the gravitational radius (GM/c^2), the particle rest mass and the rest mass of a collapsing object on the whole can really change. As a matter of fact this is just the statement of the paper by Baryshev and Sokolov (1984). The decrease of the rest mass of a gravitating body in GD follows just the conservation of energy at such a spherically-symmetric collapse, since now (unlike GR) 'an evacuation' of energy is permissible in principle in the form of scalar longitudinal waves. Thus, the possibility in principle of the change of the rest mass of gravitationally interacting bodies $\frac{\partial}{\partial x^k}$ $(\mu\, dx^k/dt)_{,k} \neq 0$ can be a basic feature of the nonlinear GD, distinguishing it from the linear approximation, where the law of mass conservation (5) was fulfilled and gravitating 'points' were assumed to be really point structureless objects.

The description of the collapse nonstationary process itself as the transition of the system in a bound state is however a nontrivial problem. In the total extent this problem requires calculating effects of the falling matter brake by radiation arising at the collapse. In accordance with the foregoing, the particle rest mass in equation (33) cannot already remain constant when falling to the centre, i.e., in general case in GD we deal with the motion of particles with the changing mass in principle.

Here, using the previous we investigate the posing of problem about results of such a collapse. First, one can attempt to study stationary stable states of the system 'particles + gravitational field' in the moments when the system does not radiate. Analogously to that, as in quantum mechanics of atom, the system is considered in stationary states or between transitions from one given energy level into another one.

Correspondingly, the *collapsar* is understood here as some stationary stable state when the following is given: 1) the total energy, 2) the dimension of a region filled by matter, 3) the total matter mass in this region, 4) the energy density of field in vacuum and its 'mass'.

The circumstance which was already mentioned in the previous section and which is to be begun with (in our opinion) is the fact that the scalar potential, i.e., the potential of repulsion found in the linear approximation of GD for the field in vacuum around a point with the mass *M,* does not change, the most probably, in the ***non***linear approximation also down to $r \approx GM/c^2$. It follows the general requirement, which will remain valid for GD also, namely that the EMT trace T of the system of interacting particles comes to the EMT trace of particles only (Landau and Lifshitz, 1973). Such a requirement is satisfied indeed precisely for the gravitational field itself and for electromagnetic classical ('massless') fields also.

In any case we use the noted circumstance and in the following we shall proceed as far as possible from the requirement: for spherically-symmetric gravitational field in vacuum to be static down to a distance of order of GM/c^2 from the centre it is necessary that two conditions for the EMT trace of the system 'particles + field' should be satisfied. On the one hand (the 'external' integral),

$$\int_{V_0} T\mathrm{d}V = \int_{V_0} Mc^2\delta(\mathbf{r})\mathrm{d}V = Mc^2 \quad , \tag{51a}$$

where

$$V_0 > (GM/c^2)^3\ 4/3\ \pi \approx 4/3\ \pi\ r_x^{\ 3} \quad .$$

And on the other hand (the 'internal' integral),

$$\int_{V_0} TdV = 4\pi \int_0^{r_x \approx GM/c^2} \mu^* c^2 \sqrt{1 - \mathrm{v}^2/c^2}\, r^2 dr = Mc^2 \quad . \tag{51b}$$

From the spherical symmetry it follows that the functions μ^* and v^2 depend on r only.

The discrete description accepted in GD holds that the density of point particles with rest masses m_a^* bound in a sphere of radius r_x can be represented in the form $\mu^* = \Sigma_\alpha\ m_a^*\ \delta(r - r_a)$. Then conditions (51a,b) must be understood in the following way: a 'point' source of scalar field located in the origin of coordinates (i.e., somewhere in the centre of a large sphere of volume V_0 (51a)) is determined in point of fact by the trace of the interacting point particles EMT located in the small sphere of radius of order of GM/c^2 (in the 'internal' integral 51b).

In a sense here we answered the question what is this collapsar mass. In point of fact, conditions (51a,b) can serve as a definition of the rest mass of supposed spherically symmetric compact configuration with the dimension of the region filled by matter of order of gravitational radius. However, here the parameter Mc^2 describes here the system 'matter + field' on the whole, defining by the conditions (51a,b) the static character of the outer gravitational field of the configuration. The mass of the compact object in the centre (the collapsar itself) will be discussed more below.

One can suppose that conditions (51a,b) will be broken in some way at the decreasing of the dimension of the sphere filled by matter down to $r \approx 0$, without excluding in principle its quantum dimension. But we shall do our best below to adhere consistently to the idea that one can always speak about interacting point particles inside the sphere $r_x \approx GM/c^2$. We shall always assume that inside the sphere of $r = r_x$ there are particles with the rest masses $m_a^* \neq 0$, interacting by means of some massless fields, or inside the sphere $r = r_x$ there is the matter with characteristic equations of state with the rigidity less than $p = \varepsilon/3$ (cf. Landau and Lifshitz, 1973). Ultimately, the question(s) formulated in Introduction comes to the question of what limits the conditions (51a,b) could be assumed consistent with the requirement that the spherically symmetric field in vacuum should remain static.

Now everything is ready for a rigorous proof of the article basic statement, i.e., of the theorem on the collapsar static field. But before going directly to the formulation of this statement I note here that I shall not touch upon uncertainties connected with the notion of an interacting point of the quantum theory of field. That is why the characteristic dimension under consideration GM/c^2 is assumed to be quite macroscopic *non*-quantum dimension for a while. Accordingly, for M values everywhere, far and wide I mean masses of the order of stellar one and more, up to cosmological masses.

Besides, one must always keep in mind that within the bounds of GD we deal in result with an idealized situation. Surely, real 'points' interact not only gravitationally. In particular, at a small distance between these points non-gravitational forces can arise which one could neglect at the distances >> Gm_a/c^2. It can be especially important at small GM/c^2 ~/>1 km. The properties of the points at the distances of the order of Gm_a/c^2 apart can completely change if to recall that the rest mass can change ($m_a \neq m_a^*$) at the 'strong' gravitational interaction or at the compression of the system down to the $r \sim GM/c^2$.

Thus, we chose conditions (51a,b) as the basic conditions determining the static character of field in vacuum. It means that scalar source (25) in the right-hand side of equation (23) remains *always the same* when approaching the centre from $r >> GM/c^2$ down to $r \approx GM/c^2$. Now it turns out that to answer the question on the static field of the collapsar - a compact configuration (see the Introduction) - one does not need at all to solve equations (49) in all their approximations. For it is sufficient to know that the potential $\Psi(r)$ and the energy density $\theta^{00}{}_{(0)}(r)$ are found already *exactly* down to $r_x \approx GM/c^2$.

Let (just for a while) dimensions of the region filled up by point particles to be unlimited from below and let for the present the energy, distributed in space out of the region, to be only the positive energy of gravitational field in vacuum - i.e.,

$$\theta_{00} = \theta^{(0)}{}_{00} + \theta^{(2)}{}_{00} \geq 0\,, \quad \theta^{(2)}{}_{00} \geq 0\ . \tag{52}$$

How close can one approach the centre for the gravitational field in vacuum to remain static? For static scalar field in vacuum we have (in all approximations) the 'scalar' energy density equal to

$$\theta_{00}^{(0)} = \frac{1}{16\pi}\frac{GM^2}{r^4}.$$

Consequently, one may write directly the integral

$$\int\limits_{(vacuum)} \theta_{00}^{(0)} = 4\pi \int\limits_{1/4GM/c^2}^{\infty} \frac{1}{16\pi}\frac{GM^2}{r^4} r^2 dr = Mc^2 . \tag{53}$$

It is evident that this integral must also account for the energy density of the gravitational field tensor component $\theta^{(2)}{}_{00}$ which in all next approximations at $r \rightarrow GM/c^2$ can differ, generally speaking, in some way from (47). And above all, this energy must be positive (non-negative) right along. So then the integral for the sum

$$\int\limits_{(vacuum)} (\theta_{00}^{(0)} + \theta_{00}^{(2)}) dV = 4\pi \int\limits_{r_x}^{\infty} (\frac{1}{16\pi}\frac{GM^2}{r^4} + \theta_{00}^{(2)}) r^2 dr$$

is equal to the same value Mc^2 at some r_x which is anyway greater than¼ GM/c^2.

Out of dependence upon the concrete form of $\theta^{(2)}{}_{00} \geq 0$, if the energy of both gravitational field and each of its components separately is positive, then before the sphere r =¼ GM/c^2 will be reached (from infinity to ¼ GM/c^2), the total energy Mc^2 of the whole configuration must already be the energy of gravitational field solely in vacuum ($\theta^m{}_m \equiv 0$). It means that even over the sphere r = ¼ GM/c^2 the scalar source (the EMT trace of the whole system) must already go to zero:

$$T = \mu^* c^2 \sqrt{1 - \mathrm{v}^2/c^2} \rightarrow 0$$

(i.e., T=0 everywhere if $\theta^m{}_m \equiv 0$). But then either velocities of all particles must become equal to the velocity of light, or the rest masses in μ^* must become zero, which is all the same. In other words, then the equations of field can be only wave equations all over the place. And the scalar source vanishes everywhere. Consequently, in that case one may not speak about any static field at all. (Of course, then there is no any newtonian limit for the gravitational field.)

Now one may say that if stable configurations with the static field outside the region filled up by particles are possible, then the dimensions of the region are anyway greater than 2 · ¼ GM/c^2. And consequently, the stationary object in the centre generating the static field may occupy only a finite volume - the sphere of a diameter *greater* than ½ GM/c^2. Hence, the impossibility follows of an infinite mean density inside this sphere, also as in principle a possibility is excluded exactly of approach (the compression) to the centre of the gravitating object down to $r = 0$.

Thus it was shown that from the fact of the positive gravitational field energy ($\theta^{00} \geq 0$) and its 'masslessness' ($\theta^m{}_m \equiv 0$) it follows that it is impossible to compress particles (matter) into the sphere of the volume less than 4/3 π(¼ GM/c^2)^3 having spherically-symmetric static field in vacuum (the tensor source trace T in (20*) not to be equal to zero). Conditions (51a,b) can be assumed consistent with the static character of the spherically symmetric field in vacuum, if an upper limit in integral (51b) is anyhow *greater* than ¼ GM/c^2.

It is natural that the energy solely of static gravitational field generated by a central object in vacuum can anyhow only be less than the total energy Mc^2 of the whole spherically symmetric configuration:

$$4\pi\int_{r_x}^{\infty}(\theta_{00}^{(0)}+\theta_{00}^{(2)})r^2dr \;<\; Mc^2 \quad \text{(in vacuum, for } r_x > ¼\, GM/c^2) \quad . \tag{54}$$

This inequality can only be strict. Correspondingly, the compact object in the centre (somewhere inside the sphere of the radius $r\sim GM/c^2$) must have a finite 'rest mass' M^*. And the rest mass M^* of the object near the centre (the collapsar proper) cannot be equal to M and must be less than M. It is possible that some part of the configuration total energy (mass) must be purely gravitational.

Now one can write the statement (the theorem) expressed above about the collapsar static spherically-symmetric field in the form of the integral condition

$$4\pi\int_{r_x}^{\infty}\left(\frac{1}{16\pi}\frac{GM^2}{r^4}+\theta_{(2)}^{00}+\theta_{*}^{00}\right)r^2dr \;<\; Mc^2 \quad , r_x > ¼\, GM/c^2 \quad , \tag{55}$$

where $\theta^{00}{}_{(2)} \geq 0$, $\theta^{00}{}_{*} \geq 0$· Here we allow a possible contribution of the positive energy of some non-gravitational (but massless) field $\theta^{00}{}_{*}$. At fulfillment of condition (51a,b), condition (55) completely solves the singularity problem in GD.

7. Conclusions

The previous has shown that in GD collapsar properties must differ from the properties of 'black holes' (BH) in GR. Though it should be said that the situation with the collapsar in GD somewhat resembles the situation with BH in GR.

In a sense, the field of GR may not be assumed static near the Schwarzschild radius. It becomes particularly clear when it is necessary 'to sew' together outside and inside solutions for the field... In GR they say about a 'solidifying', infinitely lasting collapse in the frame of reference of a remote observer. In this frame of reference the field in vacuum (Schwarzschild field) is nevertheless assumed to be static. The 'remote observer' frame of reference in GR coincides in point of fact with one determined in Section 4, in which the gravitating body is at rest in the origin of coordinates. But, it is in this frame of reference that we have shown absence of any singularity in GD within the bounds of the dynamic interpretation of 'old' equations (14).

The general statement expressed in the paper on the spherically symmetric static field following the axioms lying at the base of the relativistic field theory must greatly help concretization of the collapsar (the compact and stationary, bound object) properties in GD. In particular, it is possible that the collapsar has nevertheless a surface and its properties do not coincide completely with properties of BH described by the known Schwarzschild-Tolmen solution. In this connection it might be more correct to say not 'BH' meaning the solidifying collapse, but more carefully to call the corresponding state 'collapsar' and to call basic parameter determining such an object not Schwarzschild radius, but according to its energetic definition in the GD to call the value GM/c^2 a gravitational radius.

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