Two-Group Least Squares Method for Space Geodesy Techniques

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Abstract. Astrometric VLBI observations, laser satellite ranging and GNSS measurements for decades are used in the fundamental and applied research. The paper reviews the adaptation of the two-group Least Squares Method for operational processing of large amounts of observations (up to 10 million of GNSS) per day. The advantages of the proposed method and features of the parameterization are showed.

Keywords: space vehicles

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1 Two-Group Least Squares Method

The two-group least squares method (LSM) is described in detail in monograph Gubanov (1997). The idea of the method is that the parameters are divided into two classes: the global, common for all observations, and local, which are independently defined for each group of observations.

Let's consider some series of M group of observations. The vector of global parameters $\mathbf{x}$, the vector of parameters specific for each k-th group $\mathbf{y}_k$. In this case the data model gets the following form:

$$\mathbf{l}_k = \mathbf{A}_k\mathbf{x} + \mathbf{B}_k\mathbf{y}_k + \mathbf{r}_k \tag{1}$$

here the matrices $\mathbf{A}_k$ and $\mathbf{B}_k$ consist of partials w.r.t. global and local parameters, $\mathbf{l}_k, \mathbf{r}_k$ — observations and residuals vectors.

It can be shown that the least squares solution for (1) can be found as:

$$\hat{\mathbf{x}} = \left[\sum_{k=1}^{M} \mathbf{P}_k\right]^{-1} \sum_{k=1}^{N} \mathbf{h}_k, \tag{2}$$

where

$$\mathbf{G}_k = \mathbf{B}_k^\intercal \mathbf{Q}_k^{-1} \mathbf{B}_k, \quad \mathbf{F}_k = \mathbf{A}_k^\intercal \mathbf{Q}_k^{-1} \mathbf{A}_k, \quad \mathbf{g}_k = \mathbf{B}_k^\intercal \mathbf{Q}_k^{-1} \mathbf{l}_k, \quad \mathbf{H}_k = \mathbf{A}_k^\intercal \mathbf{Q}_k^{-1} \mathbf{B}_k,$$
$$\mathbf{f}_k = \mathbf{A}_k^\intercal \mathbf{Q}_k^{-1} \mathbf{l}_k, \quad \mathbf{P}_k = \mathbf{F}_k - \mathbf{H}_k \mathbf{G}_k^{-1} \mathbf{H}_k^\intercal, \quad \mathbf{h}_k = \mathbf{f}_k - \mathbf{H}_k \mathbf{G}_k^{-1} \mathbf{g}_k, \tag{3}$$

and $\mathbf{Q}_k$ is the observational covariance matrix.

2 Implementation in GNSS-Processing

The method considered was implemented in IAA's GNSS processing software *GRAPE* (Suvorkin et al. 2015). All astronomical, geodetic and geophysical models comply with IERS conventions (Petit & Luzum 2013) and IGS recommendations for the analysis centers (http://acc.igs.orgs 2019). We use following easimated parameters setup: *Daily (global)*: Earth rotation parameters (X_p, Y_p, $\dot{X}_p$, $\dot{Y}_p$, LOD), station coordinates, initial parameters of satellite orbits, and parameters of the solar pressure model, phase ambiguities, zenith tropospheric delay (high degree polynomial, at first iterations), atmospheric gradients (linear trends), inter-system receiver bias; *Every-moment parameters (local)*: satellite and station clock biases, zenith tropospheric delay. Soft constraints are applied on the clock biases and coordinates of several stable stations.

We process global network measurements (both GLONASS and GPS) from over 150 stations daily. The total number of measurements is about 10 million, where more than 20 thousand global parameters and 600 thousand local parameters are estimated. Occasionally, we process networks with a larger number of stations, up to 240. Processing is performed on a server based on two Intel XEON III 2.7 GHz CPUs with the RAM usage up to 100GB. Processing a daily campaign takes only about 6 hours. It should be noted that for 70 stations, the program can be executed on a personal computer in less than one hour. We compare the results of our operational calculations with high-precision IGS products. Here are some results of comparing in Table 1. These comparisons show a good level of accuracy.

3 Summary

As the results show, the described method and appropriate scheme of parametrization allow to get high accuracy of results and high performance that is especially important in perspective tasks of combined processing of colocated instruments of space geodesy observations.

Table 1. IAA GNSS EOP Service products quality

Parameter	Accuracy indicator
Satellite positions in TRF	1.5–4 cm (3D-RMS)
Stations troposphere total ZPD	3–8 mm (RMS)
Stations and satellites clock biases	50–150 ps (RMS)
X_p, Y_p (on monthly time spans)	30–60 μas
LOD	20–30 μs

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